NONLINEAR FREE-SURFACE FLOW SOLVER

Achieving Accuracy of Solution

D.C. Scullen

Scullen & Tuck Pty Ltd

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Abstract

The program NFSFS can be used to solve for flow about three-dimensional bodies of arbitrary shape moving beneath an otherwise-calm free surface. This document provides guidelines for the preparation of input files in order to obtain efficiently an accurate solution.

1 Introduction

The program NFSFS ("Nonlinear Free-Surface Flow Solver") solves for steady irrotational flow of an inviscid incompressible fluid about a given body beneath a free surface under gravity. It simulates the disturbance due to the body by using a finite but large set of discrete point-source singularities located externally to the fluid domain, specifically inside the body and above the free surface, whose strengths are determined by the program. The body and free surface are represented by collocation nodes, and the Neumann boundary condition is enforced on the body boundary, while the kinematic and fully-nonlinear dynamic boundary conditions are satisfied at the free-surface nodes. Since the free-surface location is unknown a priori, a Newton-like iterative procedure is adopted, whereby an approximation to the free surface is refined successively until convergence is achieved at a quadratic rate.

For specifics of the problem description and its solution procedure within NFSFS, the reader should refer to [1] which is available via the internet.

The Users' Guide [2] provides a description of the operation of NFSFS, including explanations of the roles and formats of the various input and output files.

This document serves to provide an enhanced description of the guidelines for the location of free-surface nodes, body nodes, singularities and points at which the radiation condition is to be satisfied, as were initially discussed in [1]. It answers the two questions:

- How does one choose the locations of collocation points and singularities in order to obtain an accurate representation of the body?
- How does one choose locations of collocation points and singularities for the initial guess of the free surface in order to obtain an accurate representation of the free surface?

In the remainder of this document, Section 2 introduces guidelines that are common to both of the above questions, Section 3 discusses those aspects that are specific to the representation of the body, including a description of a search algorithm that could be implemented to obtain an accurate representation of the body, and Section 4 gives guidelines for accurate representation of the free surface. Sections 5 and 6 discuss respectively considerations important to the combination of body and free surface and to achieving solutions with highly-nonlinear free surfaces.

2 General guidelines

In cases where lateral symmetry about $y = 0$ can be assumed, it is computationally expedient to use the singularity type `three_d_source_with_image` and to represent only one half of the body and free surface. This results in a system of equations of half the size that would otherwise be the case, and a consequential reduction in computational effort by a factor of up to 8.

Where lateral symmetry cannot be used, the singularity type `three_d_source` must be used.

Singularities are best placed approximately normal to the body and free-surface collocation nodes. For the case of the free surface, location directly above the collocation nodes is a sufficient approximation.

2.1 Offset ratio

Singularities should be placed at a distance from the surface such that they not only satisfy the prescribed boundary conditions at the collocation points, but also provide a satisfactory approximation to the boundary conditions along the segments of the boundaries that lie between collocation points. In practice, an offset ratio (i.e. the ratio of distance of singularity from collocation point compared to distance of collocation point to its nearest neighbour) of about 3 is the best choice. A choice of less than 2 for the offset ratio results in an inaccurate representation between collocation points. Greater than 5 or 6 can lead to an ill-conditioned system of equations which, although likely to produce excellent agreement with the boundary conditions over the body, may lead to free-surface convergence difficulties.

The requirement that offset ratio be within this band (of approximately 2 to 6) leads to the requirement that the collocation points used in the body and free-surface representations should be distributed approximately evenly (in all directions) within any local region. That is, a surface mesh that has elements with high aspect ratio is unable to have offset ratios based upon the various sides of the elements within the desired range simultaneously, and therefore will not produce an accurate solution.

Figure 1 shows the effect of offset ratio on satisfaction of the boundary condition. Point sources influence fluid velocity in proportion to the inverse of the square of distance, so the velocity induced by a source along the segment of a boundary between collocation points varies much less using offset ratio 2 than it does using offset ratio 0.5. In addition, the component of velocity in

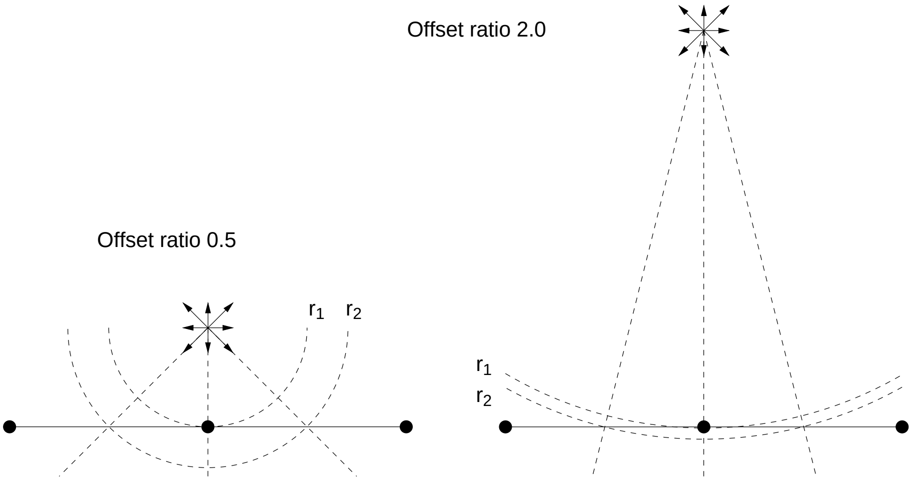

Figure 1: Diagram showing the effect of offset ratio on the accuracy of the boundary condition in the neighbourhood of a collocation point.

the direction normal to the surface is influenced by the angle between the normal and the singularity, which again varies much less with higher offset ratio. The combined result is that if a boundary condition is satisfied at a collocation point then it is well approximated along neighbouring boundary segments provided a large enough value of offset ratio is used. Here, the values of 0.5 and 2 are used for example only, to exaggerate the effect of the smaller value. As already stated, offset ratios should be in excess of 2.

There is a compromise that needs to be made between the smoother representation obtained from large offset ratio and the convergence properties of the iterative procedure which are better for smaller offset ratio. At low offset ratio, singularities influence their corresponding collocation point significantly more than any other, but at high offset ratio, singularities exert a similar influence over neighbouring collocation points. The former leads to a well-conditioned system of linear equations, the latter to an ill-conditioned system. Solutions of well-conditioned systems are characterised by small, slowly varying (in a spatial sense) singularity strengths, whereas poorly-conditioned systems produce large and oscillatory singularity strengths. The latter are likely to lead to free-surface convergence difficulties, as small changes to the free-surface approximation will produce large changes to the singularity strengths. Thus there is a practical upperbound to offset ratio, which is typically of the order of 5 or 6.

3 Representation of the body

As a general guideline for body representation, finer resolution is required wherever the flow changes direction or speed rapidly. This can be expected to be the case where the surface of the body is oblique to the direction of the vessel's motion, or where the slope of the surface (as measured with respect to the direction of motion) is large. Such regions occur at the bow and stern of the vessel and at the front and rear faces of conning towers, for example.

By comparison, little changes along the sides of vessels so, as a consequence, fine resolution is typically not required there.

3.1 Simplification of body form

In certain circumstances, it may be necessary or beneficial to simplify the form of the body.

Appendages (such as propellors) which violate the assumption of steadiness cannot be accommodated within `NFSFS`, and must be removed from the body form.

In addition, small appendages such as control fins and dive planes may not have significant affect on the flow. They are however difficult to model, and to do so correctly increases the computational burden greatly. As such, it may be best to remove them from the body.

Even more substantial components, such as conning towers, may be neglected if the component is at sufficient depth.

In evaluating the importance of existing appendages, one should attempt to compare their potential wave-making ability to that of the vessel as a whole. The factors to consider are thickness (or slope) and depth. Linearisation indicates that an object produces waves with an amplitude that is directly proportional to its thickness (and therefore longitudinal slope). (Note that doubling the thickness of a body also doubles the longitudinal slope everywhere, and results in a doubling of wave amplitude.) The dependence upon depth is more complicated, but the produced wave amplitude decays exponentially with appendage depth. For waves propagating in the direction of the vessel's motion, the relationship is $A \sim \exp(-gh/U^2)$, with the produced amplitude A of waves propagating in other directions decaying even more rapidly with appendage depth h.

3.2 Assessing the accuracy of a body representation

One method for determining if the body representation is accurate is to solve for the flow about the body alone in an infinite fluid, i.e. without a free surface. By examining the component of velocity normal to the body's surface at locations other than where the boundary condition is enforced, one may measure the level of overall satisfaction of the Neumann boundary condition, and hence the likely accuracy of the result.

Because there is no free surface, the solution is obtained without iteration. This makes determining an accurate representation of the body by an optimisation search a relatively expedient exercise.

The only necessary input files are `body.dat` and `guess_singularities.dat`.

If the file `constants.dat` is not supplied, then the vessel's speed is assumed to be unity. Flows for all vessel speeds are dynamically similar, with the velocity potential, singularity strengths and fluid velocities all scaling in proportional to that speed. Gravity does not play any role. Hence the file `constants.dat` can be omitted without loss of generality.

If the file `relaxation_factors.dat` is present, only its first entry (on the second line), which must then be unity, will be used. (This is assumed if the file is not present.)

No other input files (except `partition_size.dat`, which has no effect on the output) are used to determine the singularity strengths which satisfy the Neumann boundary condition at the body's collocation nodes.

An accurate solution will have been found only if the Neumann boundary condition is satisfied well everywhere on the body's surface, and not just at the collocation nodes where it is, by construction, satisfied exactly.

The file `velocity_components.dat` can contain coordinates of points together with directions such that, for each point, the velocity in the specified direction is to be computed. In the current context, the points would lie on the body and the directions would be normal to its surface, and would typically be taken from a representation of the body with a finer resolution than that for which the flow was solved.

The component of velocity at those locations and in those directions is reported in the file `velocity_residuals.dat`. In the current context, this is the component of velocity normal to the body surface, which would ideally be zero, and should have a magnitude very much less than that of the vessel's speed. Large flows (of the order of the vessel's speed) normal to the body surface indicate that the overall accuracy of the solution is likely to be poor, and that either the singularities should be located elsewhere or that more body collocation points should be used in the vicinity.

3.3 Improving upon the body representation

The simplest method for improving the accuracy of a body's representation is to increase the number of collocation points n being used to represent the body. Although this may be necessary, it increases the computational requirements of the solution process (which is of order at least n^2), and is therefore undesirable.

The preferred alternative is to determine the locations of the singularities which produce an acceptable level of satisfaction of the boundary conditions.

It is possible to treat the magnitude of the velocity residuals as the value of an objective function which is to be minimised by choice of location of singularities, and to implement a search method which achieves the optimisation.

In principle, each singularity can be located anywhere in three-dimensional space, so there are $3n$ parameters which control the optimisation for n singularities. For n large enough for the solution to be of acceptable accuracy, this is prohibitively expensive from a computational point of view.

It is preferable to reduce the number of controlling parameters. Since, as a general rule it is best to have singularities located approximately normal to collocation points, there is a straightforward reduction to n parameters, being the distances along each of those normals. This is probably still too large to be of any great benefit though.

The extreme alternative is to have only one controlling parameter, being the ratio of singularity offset to local mesh spacing. This, coupled with increasing mesh resolution where velocity residuals are large, is likely to be a good compromise between expending too much effort in determining the optimum location of the singularities and expending too much effort in solving the final flow due to unnecessarily-high body resolution.

Once the body representation is found to be satisfactory, the user may then proceed to include the free surface and to solve for the resulting free-surface shape.

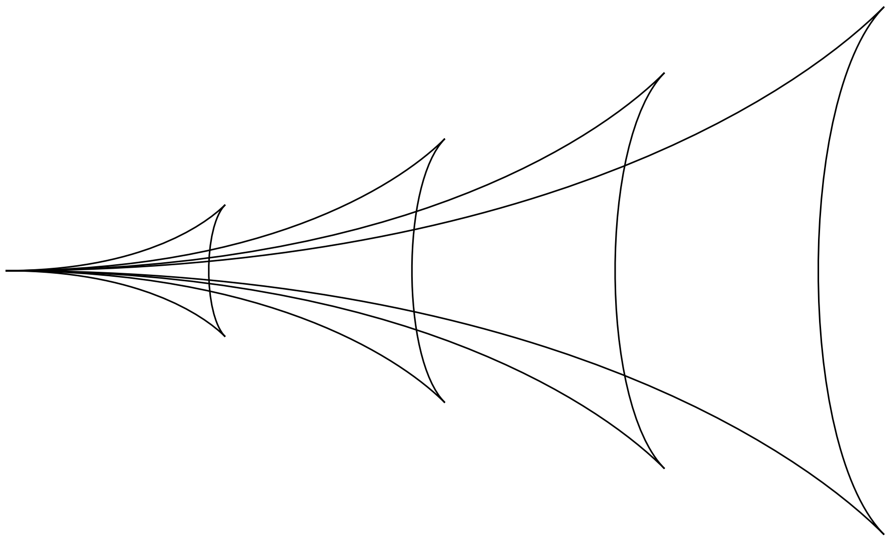

Figure 2: Schematic showing transverse and divergent waves within a typical ship wake.

4 Representation of the free surface

4.1 Ship wake characteristics

The free-surface representation is driven predominantly by the characteristics of ship wakes, so it is appropriate to give a brief description of those characteristics here.

Within linear theory, ships produce wakes which live within wedges with an angle of $2\arcsin\frac{1}{3} \approx 39°$ at their apex. Accordingly, the wake envelope is approximately two-thirds as wide as it is long.

The wave pattern within that envelope consists of two different groups of waves; transverse waves which propagate essentially in the same direction as the ship itself, and divergent waves which propagate essentially sidewards from the centreline of the ship's path. Figure 2 shows these wave groups schematically, confined within the wedge with apex angle approximately 39°.

Slowly-moving or large ships produce wave patterns that consist primarily of transverse waves. The divergent waves are still present, but their amplitude is smaller by comparison. High-speed or small vessels (e.g. speedboats) produce wave patterns that consist primarily of divergent waves. The transverse waves are still present, but their diminished amplitude coupled with their very-much-longer wavelength make them difficult to discern.

Wavelength is determined by the speed of the ship, and is proportional to its square. Those waves that propagate directly in line with the ship (i.e. at an angle of $\theta = 0$) have a (fundamental) wavelength of $\lambda_0 = 2\pi U^2/g$. Waves propagating in other directions θ have wavelength $\lambda = \lambda_0 \cos^2 \theta$. Clearly this reduces to zero as $\theta \to \pi/2$.

In practice, we are interested in the steady flow through calm water. It is possible for the sea surface to be steady when observed from a frame of reference moving with the ship, even if the sea itself has ambient waves. That ambient sea must necessarily have waves that move only in the direction of the ship and with the same speed (and therefore have wavelength λ_0), but those waves may have arbitrary amplitude and phase. We are interested only in the particular case when that amplitude is zero.

4.2 Free-surface representation

In principle, the free-surface computational domain can be any shape, but in practice a rectangular domain is most useful (even if only from the point of view of graphical presentation).

The domain must extend far enough ahead of the body to allow successful implementation of the radiation condition, which enforces that there be no waves ahead of the ship, i.e. that the ambient sea has no waves. Typically, a domain that extends one or two fundamental wavelengths ahead will suffice. If the domain is truncated too close to the body, then the solution will contain small-amplitude incident waves, indicating violation of the radiation condition.

The domain must extend sufficiently far aft of the body to allow successful representation of the resulting waves. Typically, 4 or 5 fundamental wavelengths is sufficient. If the domain is truncated too close to the body, then the solution may be inaccurate at the rear boundary.

If the domain extends too far aft, convergence difficulties may be encountered due to the shortening of the wavelength between iterations. This is a nonlinear effect which becomes more pronounced as wave amplitude is increased. Then, small changes in wavelength can produce large changes in free-surface location far aft of the body, and this may affect the convergence of the iterative procedure.

The domain should be wide enough to encompass the resulting wave pattern. It should therefore have a width approximately two-thirds of the distance between the bow of the body and the aft end of the domain. If symmetry is being enforced by the use of singularities of the type `three_d_source_with_image` then only one lateral half of the wake needs to be represented, and the width can be halved.

The free-surface domain should be longer and wider than the body about which the flow is being determined.

The free-surface mesh should have rectangular elements with aspect ratio near one, i.e. the mesh elements should be approximately square.

The free-surface resolution should be such that the dominant waves are resolved well. Typically, at least 8 collocation nodes are required per wavelength for transverse waves. High-speed flows, for which the wave patterns are strongly divergent, will be difficult to solve accurately, since resolving the divergent waves well requires high lateral resolution, which in turn requires high longitudinal resolution.

The radiation condition should be enforced at two of the foremost rows. This involves adding the index number of each of the collocation points in those rows to the file `radiation_points.dat`. Physically, the radiation condition selects the member which has no ambient waves from the set of all possible steady-state solutions (which are those with an ambient sea as described above, with arbitrary phase and amplitude). Mathematically, this is done by applying conditions to enforce that the ambient-sea elevation be small, and is best applied at two rows which are of the order of a quarter of the fundamental wavelength apart. If 8 collocation nodes are used to represent a transverse wavelength, then this implies that the two rows of radiation points should be separated by a single row of collocation points, e.g. rows 1 and 3.

Since the program finds the potential by inversion of a system of linear equations, there must be the same number of singularities as the sum of the number of body nodes, surface nodes, and radiation condition nodes. Additional singularities are required, and are typically placed one row ahead of the free surface and one row aft.

Singularities should be located above the free surface, at a height sufficient that they will always be above the highest wave generated. (Wave amplitude cannot exceed the stagnation height of $U^2/(2g)$.) A reasonable guideline is a distance of approximately 3 to 4 times the local free-surface grid size.

Too fine a free-surface resolution may produce difficulties, as singularities

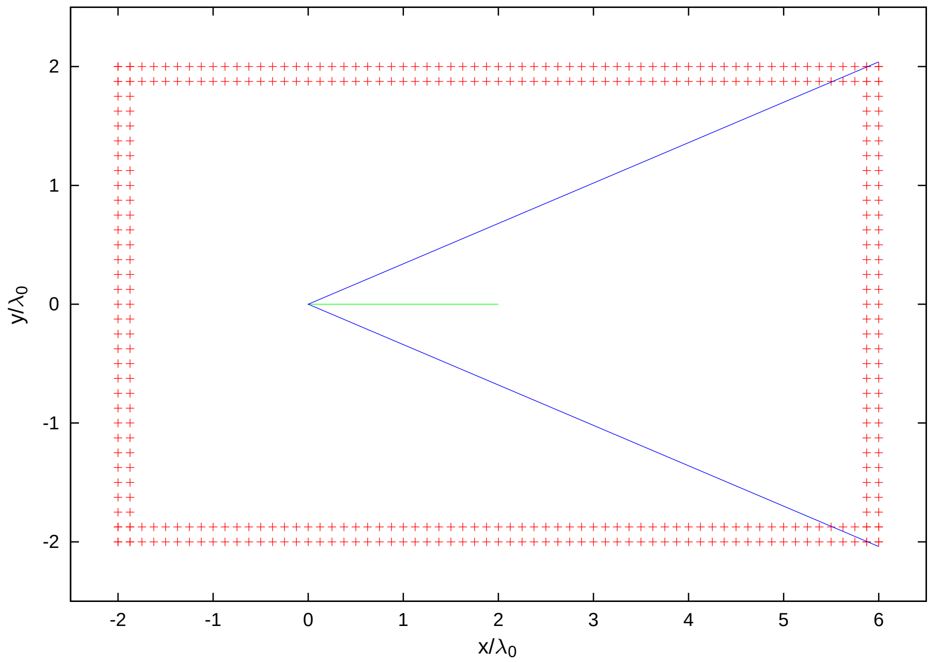

Figure 3: Layout of free-surface collocation points, showing length, width and resolution of domain, together with location of body and wake wedge.

may then be required to be too close to the initial guess free surface, and may enter the fluid domain during subsequent iterations as the free-surface approximation is refined.

Figure 3 shows a typical layout for the free-surface collocation points, with lengths scaled relative to the fundamental wavelength. Only the outermost rows and columns of collocation points are displayed. The solid lines represent the approximate location of the body and the wedge within which the wake is contained. Note that the computational domain extends ahead of the body by about two fundamental wavelengths, aft of the body by about four and to the sides by about two. The longitudinal resolution is such that there are 8 collocation points per fundamental wavelength; the lateral resolution is chosen to match the longitudinal resolution. Singularities will be placed directly above collocation points using an offset ratio of about 3, with an additional row of singularities ahead and behind the free-surface mesh. The radiation condition will be enforced for collocation points in rows 1 and 3. Note that for bodies exhibiting lateral symmetry, the singularity type `three_d_source_with_image` should be used, and then only one half of the free-surface computational domain is required.

5 Matching of the combination

Although generally not critical, it is, to an extent, important to match the resolutions of the body and free surface.

High-speed flows have a fundamental wavelength that is large by comparison to the body size. This suggests that accurate representation of the fine details of the body is unnecessary, and that a reasonable solution can be obtained provided that the overall characteristics of the body are captured. To that extent, efforts towards obtaining an accurate representation of the body are unnecessary, and do little more than increase the computational burden. As already mentioned however, it is computationally expensive to obtain a smooth representation of the free surface for a highly-divergent wave pattern.

In contrast, low-speed flows have a fundamental wavelength that is short compared to the size of the body. A potential consequence is that there are many transverse waves between bow and stern, each of which needs to be resolved sufficiently, and therefore many rows (and therefore many columns) of collocation points may be required. Equally, it is important that the body is resolved on a similar scale, especially if the body is shallowly submerged.

These two statements together represent limitations on the effectiveness of `NFSFS` for very-high-speed or very-low-speed flows. In practice, flows for which the Froude number $F = U/\sqrt{gL}$ (where L is the length of the body) is much closer to zero than 0.2 or greater than approximately 1 are difficult to solve accurately.

Deeply submerged bodies produce predominantly transverse wave patterns, and the fine details of the body shape have little impact upon the resulting free surface. Once again, little is to be gained (and efficiency is to be lost) from employing a high-resolution representation of the body.

6 Highly-nonlinear flows

Special care may be required in execution of `NFSFS` if the resulting flow is expected to have large-amplitude nonlinear waves. As already mentioned, an upperbound for the elevation of the sea's surface is stagnation height, $U^2/(2g)$. Waves with an amplitude exceeding half of this are significantly nonlinear.

`NFSFS` employs an iterative procedure which converges quadratically to the solution. As with all iterative procedures, there is a risk of divergence. This risk is greatest in the earliest iterations, when the guessed free surface is not a good approximation of the actual solution. The likelihood of divergence can be reduced if a relaxation factor less than one (but always greater than zero) is employed, so that `NFSFS` reduces the amount by which the velocity potential is modified between iterations. Relaxation factors are set in the file `relaxation_factors.dat`. In extreme cases, the relaxation factor could be reduced to as little as 0.1 for the first iteration, increasing gradually to unity over the next several iterations. Of course, for small-amplitude waves the relaxation factor can safely be set to one from the start, or equivalently, the file `relaxation_factors.dat` can be removed. In practice it is difficult to know in advance if a relaxation factor is to be required, and if so, how small it should be, and for how many iterations. This is simply a characteristic of any aggressive search algorithm, and there is little more science to it than trial and error. Start without relaxation, and if divergence occurs in the early iterations, introduce a modest value for a few iterations. If divergence still occurs, use smaller values for longer. This difficulty is compounded by the fact that some large-amplitude wakes produce breaking waves, which cannot be captured by `NFSFS` (or indeed any other existing program!).

A complimentary technique, which can be used in conjunction with relaxation if necessary, is to approach the desired case using solutions from a sequence of less difficult cases. For example, shallowly-submerged submarines produce large-amplitude nonlinear waves (and in extreme cases, breaking waves) which are difficult to capture. Deeply-submerged submarines are easily solved for. It is possible to use the converged free surface and singularity strengths from a more-deeply submerged submarine as an initial guess for the desired case. This can be achieved simply by renaming the files `converged_surface.dat` and `converged_singularities.dat` to `guess_surface.dat` and `guess_singularities.dat` respectively. For extreme situations, a sequence can be used, with the converged output from the previous case being used as the guess input for the next.

7 Conclusion

In summary, presented in this document are guidelines for choosing the locations of collocation points and their associated singularities for both the body and guess free surface in order to obtain an accurate solution to the nonlinear free-surface flow problem. This includes discussions of factors common to both the body and free surface, and factors particular to the body alone, to the free surface alone, and to their combination. As an important tool for obtaining an accurate representation of the body, an optimisation search based upon minimising flow through the body has been described. In addition, some discussion was devoted to the task of solving for highly-nonlinear large-amplitude waves by the use of relaxation factors, optionally combined with approaching the desired solution via a sequence of smaller-wave-producing cases.

References

[1] Scullen, D.C. *Accurate computation of nonlinear free-surface flows*. PhD Thesis, The University of Adelaide (1998) `http://www.maths.adelaide.edu.au/people/dscullen.html`

[2] Scullen, D.C. and Tuck, E.O. *Nonlinear free-surface flow solver — Users' guide*. Scullen & Tuck Pty Ltd (2002) `http://www.maths.adelaide.edu.au/people/dscullen.html`