Three-Dimensional Steady State Nonlinear Free Surface Flow Computations

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Abstract

A numerical method for computing the nonlinear steady-state three-dimensional potential flow about a body beneath an otherwise calm free surface has been developed and implemented. The method involves a distribution of point sources external to the fluid domain which provides a simple description of the potential, yet yields an accurate solution along the free surface and body, and within the fluid domain. The boundary conditions are coupled with an iterative procedure in a manner that produces quadratic convergence to the solution potential.

Results will be shown for nonlinear flow about a submerged spheroid. They will be compared quantitatively to the appropriate linear theory and other numerical computations, and some interesting nonlinear cases will be presented.

1 Introduction

The problem addressed in this paper is that of computing the three-dimensional flow about a submerged body moving at constant speed through an otherwise calm sea. Although this work is motivated by the desire to compute flows about surface-piercing objects (such as ships), the submerged body problem is of interest in its own right.

To move near a free surface in a fluid, a body must overcome both wave and viscous drag. For many cases of interest (for example, long and thin bodies) wave drag is the largest component and is a reasonable approximation of the total drag force. In these cases, if we are able to calculate the forces acting on a (somewhat arbitrarily shaped) body then we are in a position to modify the body shape in order to improve efficiency, which is of course of considerable practical interest.

By considering the irrotational flow of an inviscid and incompressible fluid, the problem is reduced to that of satisfying Laplace's equation for the velocity potential ϕ in the fluid domain subject to a Neumann boundary condition on the body and kinematic and (nonlinear) dynamic boundary conditions on the (yet to be determined) free surface. Additionally, the radiation condition stating that the fluid is not disturbed far upstream must be imposed.

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The problem is stated formally (in the frame of reference of the body) as

$$
\begin{aligned}
\nabla^2\phi &= 0 \quad \text{in the region of the fluid,} && (1)\\
\frac{\partial\phi}{\partial n} &= 0 \quad \text{on the body surface,} && (2)\\
\frac{\partial\phi}{\partial n} &= 0 \quad \text{and} && (3)\\
p &= 0 \quad \text{on the free surface} && (4)\\
\text{and} \qquad \nabla\phi &\to 0 \quad \text{far upstream} && (5)
\end{aligned}
$$

where p is the excess pressure above atmospheric.

The numerical method used to solve this problem is a straight-forward extension to three dimensions of the method used by Scullen and Tuck (1995) to solve the analogous two-dimensional problem.

Although the method is suitable in general for arbitrarily shaped submerged bodies, this paper will concentrate solely on a prolate spheroid moving in the direction of its (horizontal) axis at steady speed. In particular, the spheroid whose diameter is one fifth of its length is considered. This case has been computed previously by several others such as Doctors and Beck (1987), Cao (1991), Bertram *et al* (1991), and so provides a useful benchmark for comparison of numerical results. Also, the linear theory proposed by Havelock (1931) can be used as a guide to accuracy for cases of small disturbances.

Having fixed the ratio of diameter D to length L to be 0.2, the problem can be parameterised by two non-dimensional quantities. The Froude number F based upon length L is defined as $F = U/\sqrt{gL}$ where U is the speed of the body and g is acceleration due to gravity and can be considered as a non-dimensional speed. The ratio of diameter D to depth of submergence h of the axis of the body can be interpreted as a measure of the magnitude of the disturbance.

2 Verification of Accuracy

The linear theory for the drag experienced by a submerged spheroid was stated by Havelock (1931), and results produced by the current method are consistent with this approximation for sufficiently small disturbances. The present method shows that for larger disturbances the linear theory underpredicts the wave resistance at low Froude number, and overpredicts at high Froude number. Nevertheless, the investigation shows that the linear theory provides quite a reasonable approximation of drag over the range of Froude numbers and disturbances considered.

Additionally, the current method compares well with results produced by Doctors and Beck (1987), Cao (1991) and Bertram *et al* (1991). It is important to note however that the numerical methods employed by Doctors and Beck and Cao were time dependent formulations that were extrapolated to approximate the steady state. The method employed here, being a steady state calculation, is computationally inexpensive by comparison.

3 Analysis of Computational Results

Solutions were sought for a series of values of D/h over the range of Froude numbers from 0.20 to 1.00, in intervals of 0.05. Figure 1 shows the maximum deviation of the free surface from rest (nondimensionalised with respect to U and g) for cases in which results were obtained. Solutions were found at each of the desired Froude numbers for values of D/h up to and including 0.9, for which the depth of the top of the spheroid is approximately 0.6 of the diameter of the body, which is quite shallow. For larger disturbances (higher values of D/h), solutions at Froude numbers between 0.4 and 0.6 were not obtained. At the Froude number of 0.20, a solution was found for a D/h value of 1.6. This is a particularly shallow body, as the depth of the top of the body is only one eighth of the diameter.

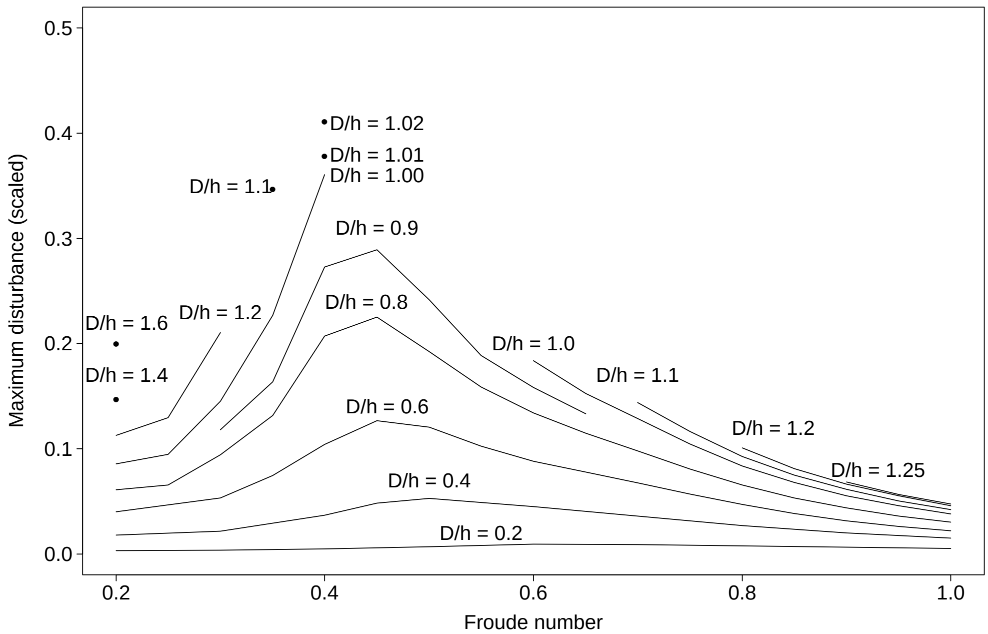

Figure 1: Maximum deviation of the free surface from undisturbed versus Froude number for various values of D/h. Amplitude is scaled with respect to U^2/g so that 0.5 is stagnation height and hence an upperbound.

Of particular interest here are the regimes in which the flow shows nonlinear characteristics, as presumably it is in these cases that the nonlinear nature of the solution method becomes advantageous. One such case is that of a body that is shallow in comparison to both its diameter and the amplitude of the waves that it creates. The spheroid at diameter to depth ratio D/h of 1.25 and Froude number of 1.0 is a suitable example of this. The top of the body is at a depth that is 0.3 of the diameter, and is roughly the same as the depth of the first trough. This can be seen more clearly in Figure 2.

Also of interest is the rapid growth in wave amplitude as the disturbance becomes large. In this regime, small changes in depth produce large changes in waveheight, the waves display significant nonlinearity as the first crest is considerably sharpened, and it can be argued that the waves are nearing an amplitude at which they would break. Such cases are shown in Figure 1 at a Froude number of 0.4, for D/h values of 1.00, 1.01 and 1.02. Note the rapid

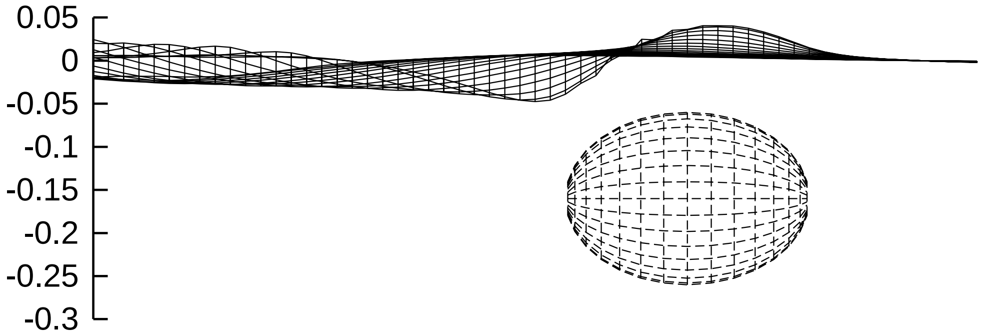

Figure 2: Side view of flow at Froude number 1.00 for $D/h = 1.25$. The top of this body is at a depth comparable with that of the first trough.

growth, and recall that in these nondimensionalised units, stagnation height is 0.5. It is likely that a wave in fact breaks before its crest reaches stagnation height. Figure (3) shows the case where the diameter to depth ratio is 1.02. Here it is reasonable to expect that only a small increase in the disturbance would result in a breaking wave.

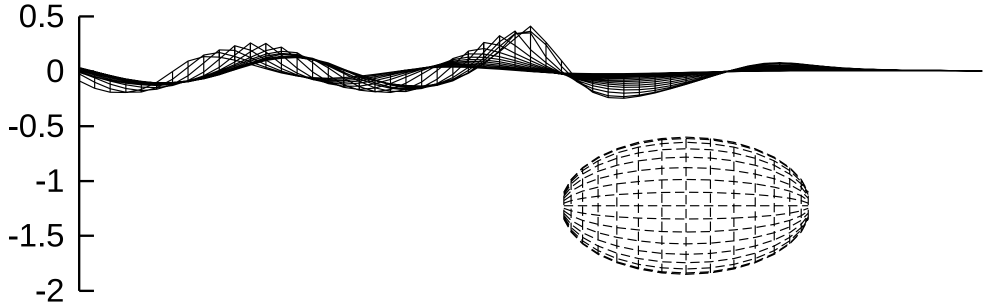

Figure 3: Steep wave produced by a spheroid with $D/h = 1.02$ at a Froude number of 0.40.

4 Conclusion

The numerical method employed here is consistent for small disturbances with the appropriate linear theory for submerged prolate spheroids, and hence the numerical accuracy is verified. This accuracy is further supported by the agreement of lift and drag forces with those calculated by other independent numerical computations. Investigations into larger disturbances show that the linear theory is generally a good indicator of wave resistance, although it has a tendency to underpredict at low Froude numbers, and overpredict at high Froude numbers. The results obtained are of greatest value when looking at cases for which the solution exhibits strong nonlinear characteristics, as accurate solutions for such cases are not obtainable from the linear theory.

The method is capable of handling arbitrarily shaped bodies, and so is able to produce accurate information about flows around bodies for which we do not have any appropriate analytic tool or theory. It will do so in a (relatively) computationally inexpensive manner.

Finally, it is anticipated that the methods employed here should prove to be suitable for surface-piercing (ship) problems.

5 References

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