SEA WAVE PATTERN EVALUATION

Part 1 Report: Primary Code and Test Results (Surface Vessels)

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Abstract

This project involves development of validated FORTRAN-90 code to compute surface elevations and velocity fields produced by steadily moving bodies either on the surface or submerged. In the two-part project agreed to for the 1998-1999 year, the waves will be computed in the far field for water of infinite depth using thin-ship theory.

Part 1 with delivery date 30 April 1999 coinciding with that of the present report has produced preliminary surface elevation code ("SWPE1.0") for surface ships. Part 2 with delivery date 31 May 1999 will extend the code to submerged bodies, and to velocity field computations, and results of validation exercises will be reported.

The present Part 1 Report gives the theoretical basis for the code, both for surface and submerged bodies, and outlines the work to be done in Part 2.

1 Ship wave theory

The steady surface wave pattern $z = \zeta(x, y)$ of any moving body, as seen at a point (x, y) sufficiently far from the ship, is of the form of a sum of plane waves travelling at various angles θ of propagation relative to the direction of motion (negative x-axis) of the body. Thus [4],[11],[8] p.277

$$\zeta(x, y) = \Re \int_{-\pi/2}^{\pi/2} A(\theta) e^{-i\Omega(\theta)}\, d\theta \tag{1}$$

where

$$\Omega(\theta) = k(\theta)\,[x\cos\theta + y\sin\theta]$$

is a phase function. Here $A(\theta)$ is the amplitude and $k(\theta)$ the wave number of the wave component travelling at angle θ. Note that the contributions to this integral from *positive* angles θ correspond to waves being propagated to the *left* or *portside* of the body. In the present report the body and flow are assumed to have lateral symmetry, and then $A(\theta)$ is an even function and need only be computed for $\theta \geq 0$.

Once $A(\theta)$ and $k(\theta)$ are specified, we can use (1) to determine the actual wave pattern. At any given speed U, the amplitude function $A(\theta)$ is a property only of the body's hull geometry, whereas $k(\theta)$ is fully determined from the dispersion relation for plane waves. In infinite water depth, which we shall assume in the present report, we have simply

$$k(\theta) = k_0 \sec^2\theta \tag{2}$$

where $k_0 = g/U^2 = k(0)$ is the wave number of pure transverse waves at $\theta = 0$.

The (complex) amplitude function $A(\theta)$, sometimes also called the free wave spectrum or Kochin function, can be computed by various means, e.g. Michell's [6] thin-ship theory, a complete nonlinear near-field computation [7],[1], or by experimental measurement [2]. For a thin body with offsets $y = Y(x, z)$ in water of infinite depth, Michell's [6] theory indicates that

$$A(\theta) = -\frac{2i}{\pi} k_0^2 \sec^4\theta \iint Y(x, z) \exp(k_0 z \sec^2\theta + i k_0 x \sec\theta)\, dx\, dz, \tag{3}$$

where the integral with respect to x is along the length of the body, and that with respect to z over the actual vertical position of the body, whether or not submerged. If one wishes to use fixed offsets $Y_0(x, z)$ for a body with variable depth of submergence h, then $Y(x, z) = Y_0(x, z + h)$ in (3), so that (3) becomes

$$A(\theta) = -\frac{2i}{\pi} k_0^2 \sec^4\theta \iint Y_0(x, z) e^{-k_0 h \sec^2\theta} \exp(k_0 z \sec^2\theta + i k_0 x \sec\theta)\, dx\, dz, \tag{4}$$

where the integral with respect to z is now taken over the (fixed) draft of the (unsubmerged) body. The above formulae can also be generalised (see [5, 12]) to multihull vessels.

Although it is not the topic of the present report, the total energy left behind in the wave field can easily be computed once the amplitude function $A(\theta)$ is known, which leads to the famous Michell integral for the wave resistance R for a ship, namely

$$R = \frac{\pi}{2}\rho U^2 \int_{-\pi/2}^{\pi/2} |A(\theta)|^2 \cos^3\theta \, d\theta \, . \tag{5}$$

In the present report, instead of working out the integral (5) for R, we substitute the integral (1) for $\zeta(x, y)$. The final θ-integration is a considerably harder computational task than that for R, and involves more tabulation as it depends on the spatial coordinates x, y. However, the task retains some similarities to that for the wave resistance, and experience with wave resistance codes is valuable.

The thin-ship theory of Michell represents the body by a centreplane source distribution proportional to its longitudinal rate of change of thickness (local beam). The only requirement for its validity is that that quantity be small. Hence the theory applies as well to submerged as surface-piercing bodies. In some cases especially for submerged bodies it is possible instead of thin-ship theory to use the somewhat simpler "slender-body theory", where the body is represented by a line of sources rather than a plane distribution, but we shall not use slender-body theory in the present report. In any case, the thin-ship theory includes the slender-body theory, and specifically gives the latter as a limit of the former for large beam/draft ratios. In general, there is no restriction on beam/draft ratio for validity of thin-ship theory, so long as the beam/length ratio is small. For example, thin-ship theory applies perfectly well to submerged slender bodies of revolution.

2 Computation of wave amplitude function

The double integral (3) over the body's centreplane, determining $A(\theta)$ for each fixed angle θ from the offsets $Y(x, z)$, can be reduced to a pair of separate integrals over depth and length as follows.

First evaluate for all stations x the integral

$$F(x, \theta) = \int Y(x, z) \exp(k_0 z \sec^2\theta) \, dz \tag{6}$$

where $k_0 = g/U^2$, and the integral is in the vertical z-direction, from the lowest point of the section in its actual position with $z < 0$, up to the waterline $z = 0$. If the body is submerged to mean depth h, either there are artificial offsets of zero width placed between its top and the surface, or better, we replace $Y(x,z)$ by $Y_0(x,z)\exp(-k_0 h \sec^2\theta)$ and integrate over the unsubmerged range of z-values. The standard offsets $Y_0(x,z)$, namely half-widths of the body at station x and waterline z, are specified as data.

Next, evaluate simultaneously a pair of integrals along the body from bow to stern, namely

$$P(\theta) = \int F(x,\theta)\cos(k_0 x \sec\theta)\,dx \tag{7}$$

with a similar integral for $Q(\theta)$ having sin( ) instead of cos( ). For fore-aft symmetric bodies, we have $Q = 0$, but we do not make that assumption in general. This integral takes as input the quantity F just evaluated.

Then equation (3) states that

$$A(\theta) = -\frac{2i}{\pi}k_0^2 \sec^4\theta\,[P(\theta) + iQ(\theta)]\ . \tag{8}$$

The above is only a slight variation from standard versions of the original Michell integral, an integration by parts having been used to express the x-integral in terms of the actual offset Y rather than its longitudinal derivative Y_x. It is during this integration by parts that the assumption is made that the offsets vanish at both ends, which rules out "transom sterns". However, in future versions of the program, this limitation will be removed.

There have been few attempts to set up standard routines to evaluate these integrals from input offset data. Perhaps for conventional ships the incentive has been absent, since Michell's theory has (unfairly) had a bad press. For especially thin hulls, there is less reason to doubt the validity of the theory, and hence greater potential value in setting up such routines. In any case, one of the objectives of Part 2 of the present project is to test the quantitative validity of the thin-ship approximation.

Most efforts on computing Michell's integral have been for mathematically defined hulls, for which one, usually both, of the above integrals can be evaluated analytically. Although such idealised cases are useful to test the program, we assume here that all integrals must be done numerically using input data for the actual offsets $Y(x,z)$.

In principle, the above numerical integration tasks are quite straightforward, and could even be done using standard (e.g. IMSL) packages. However, there are a number of special features, especially the singularity as $\theta \to \pi/2$. In that limit (which corresponds to the diverging part of the ship wave pattern), the effective wave number $k_0 \sec\theta$ becomes infinite, and the x integral has a highly-oscillatory integrand. This is a well-understood problem, but it does require some care, especially in data specification near the bow and stern, which contribute most to the integrals in that limit. It is moderated, especially for submerged or semisubmerged hulls, by the exponential decay factor from the z integral that damps out contributions from near $\theta = \pi/2$.

The present implementation uses a special trapezoidal-like algorithm for the z integral and Filon's quadrature for the x integral. Filon's quadrature [3] is a procedure with the same accuracy as Simpson's rule that can account for rapidly-oscillating integrands. Specifically, it is exact whenever the function F appearing in the x integral is piecewise quadratic. If instead Simpson's rule was used for the x integral, this would be exact when the whole integrand (i.e. F times a cosine or sine) was piecewise quadratic. If there is no contribution from the extreme bow and stern (i.e. $Y = 0$ and hence $F = 0$ there), the algorithm used is simply

$$P(\theta) \approx \sum_{i=1}^{N_x-1} \omega_i F(x_i, \theta) \cos(k_0 x_i \sec\theta)\, \Delta x$$

where the length of the ship has been divided into N_x equal segments each of length Δx, i.e. there are $N_x - 1$ stations $x = x_i$, $i = 1, 2, ..., N_x - 1$, not counting the extreme bow $x = x_0$ or stern $x = x_{N_x}$. The even weights are

$$\omega_{2i} = (3K + K\cos 2K - 2\sin 2K)/K^3$$

for all i, and the odd weights are

$$\omega_{2i+1} = 4(\sin K - K\cos K)/K^3$$

for all i, where $K = k_0 \sec\theta \Delta x$. When K is small, these weights tend to the standard Simpson values ($\omega_i = 2/3$ even, $4/3$ odd), so, as expected, the influence of the Filon method is felt mainly for large K, i.e. as $\theta \to \pi/2$. Note that if transom sterns are allowed, there are also special Filon weights associated with the extreme stern offsets at $i = N_x$. These weights are included in the preliminary (Part 1) version of the program, but the transom stern modifications are not documented or tested at this stage.

The core of the implementation is the z integral. This has an integrand consisting of the input offset Y times an exponential decay factor (noting that z is always negative here). The latter causes little difficulty, and the ordinary trapezoidal or Simpson rule could have been used. However, there are potential advantages in an alternative Filon-like procedure (c.f. [10]) for which the answer is exact when Y itself, rather than Y times the exponential, is piecewise smooth. We assume it is piecewise linear, which gives trapezoidal-rule accuracy.

The actual algorithm used is

$$F(x,\theta) \approx \sum_{j=0}^{N_z} \omega_j Y(x,z_j) \exp(k_0 z_j \sec^2\theta)\Delta z$$

where the section is divided in the vertical into N_z segments each of height Δz, with end-points z_j, $j = 0, 1, 2, ..., N_z$, and the weights ω_j are given by

$$\begin{aligned} \omega_0 &= \left(e^K - 1 - K\right)/K^2 \\ \omega_{N_z} &= \left(e^{-K} - 1 + K\right)/K^2 \end{aligned}$$

and for $j \neq 0, N_z$,

$$\omega_j = \left(e^K + e^{-K} - 2\right)/K^2$$

where

$$K = k_0 \sec^2\theta\Delta z \ .$$

(Note that these are different definitions of K and ω's from those for the above x-wise Filon quadrature). These weights reduce to those of the ordinary trapezoidal rule ($\omega_0 = \omega_{N_z} = 1/2$, $\omega_j = 1$ otherwise) when $K \to 0$, so the impact of the special algorithm is felt most for large K, i.e. again when $\theta \to \pi/2$. If such a provision is not made, unless the grid size Δz is very small, the correct rate of decay of the diverging wave contribution is not captured.

Although, since the z integral is independently evaluated for each separate value of x, and therefore we could choose a different grid for every station, it is convenient to use a universal set of waterplanes $z = z_j$ that is the same for all stations, and is specified in the input offset data. For semi-submerged hulls where the lowest point of the hull is higher near the bow and stern than at mid-section, this means that we must input and use in the above algorithm a certain number of artificial zero offsets. In that case $\Delta z = D/N_z$ where D is the draft at the deepest point of the hull, usually at midship.

Note that a feature of this implementation is that the z integral is exact for rectangular sectioned struts. Since the Filon rule is similarly exact for parabolic x-variation, the program yields identical output for parabolic struts to that of check routines that do the z and x integrals analytically.

3 The θ integral

For the wave resistance integral (5) with respect to θ, we can just use Simpson's rule. So long as we have used a Filon-like procedure in computing $A(\theta)$ to ensure accuracy and convergence as $\theta \to \pi/2$, there is no further complication in the subsequent θ-integration, the integrand of (5) being a smooth and relatively slowly varying function of θ. Hence Simpson's rule with (say) 100 θ-values works adequately and yields about 4-figure accuracy in most cases.

However, this is not the case for the wave field integral (1), especially in the far field. The issue is again one of integration of rapidly oscillating functions, and now such rapid oscillations occur at (almost) all θ values when x and y are large. Indeed, in the limit as $r = \sqrt{x^2 + y^2} \to \infty$, the method of stationary phase can be used to show that contributions to (1) come only from near two special values of θ where such oscillations are minimised. One computational procedure would be simply to evaluate these stationary contributions, obtainable explicitly from Wehausen and Laitone [13],p. 487.

However, there are considerable objections to such a procedure, not only the obvious objection that it applies only for extremely large r. The formal stationary phase asymptotic expressions become singular along the Kelvin lines $y = \pm x \arctan(1/3)$, are exactly zero outside the Kelvin angle, and have some undesirable features inside it.

On the other hand, the full (single) integral (1) is valid reasonably close to the body, and gives a uniformly valid and non-singular result for all $x > 0$, providing we can evaluate the integral numerically with accuracy. This can be done by Simpson's rule, as with the wave resistance integral (5) so long as x and y are not too large, say $k_0 r < 5$. Beyond that distance, the number of θ values required becomes excessive (more than 100,000).

A procedure for maintaining numerical accuracy as r increases is outlined in [11], Appendix A. This procedure attenuates the integrand for those θ values where it is highly oscillatory, but otherwise uses conventional numerical quadrature on a uniform mesh. The attenuation is applied as an exponential "fade factor", where the integrand is unchanged near to the two θ values of stationary phase, but decays smoothly as we move away from those θ values.

This allows accurate results to be obtained on a much coarser grid, measured in hundreds of θ values instead of hundreds of thousands. It is possible that even better procedures could be devised (e.g. use of a non-uniform grid with mesh elements concentrated near the stationary values), but this is left for future work.

4 Velocity potential and flow fields

The wave elevation ζ given by (1) is accompanied by a flow field with velocity potential $\phi(x, y, z)$ such that $\zeta(x, y) = -(U/g)\phi_x(x, y, 0)$. For infinite water depth, ϕ is given in $z \leq 0$ by

$$\phi(x, y, z) = -\Re iU \int_{-\pi/2}^{\pi/2} A(\theta) e^{k_0 z \sec^2\theta} e^{-i\Omega(\theta)} \cos\theta \, d\theta \; . \qquad (9)$$

Thus the flow velocity components are (u, v, w) where

$$u(x, y, z) = -\frac{g}{U}\Re \int_{-\pi/2}^{\pi/2} A(\theta) e^{k_0 z \sec^2\theta} e^{-i\Omega(\theta)} \, d\theta$$

$$v(x, y, z) = -\frac{g}{U}\Re \int_{-\pi/2}^{\pi/2} A(\theta) e^{k_0 z \sec^2\theta} e^{-i\Omega(\theta)} \tan\theta \, d\theta$$

and

$$w(x, y, z) = -\frac{g}{U}\Re i \int_{-\pi/2}^{\pi/2} A(\theta) e^{k_0 z \sec^2\theta} e^{-i\Omega(\theta)} \sec\theta \, d\theta \; .$$

The code will be extended in Part 2 to include output of (u, v, w) or ϕ as well as ζ. The new data will be available on the surface $z = 0$ as well as at appropriate depths $-z$, and will be further discussed in the 31 May 1999 report.

5 Validation by comparison with the literature

A literature search will be performed in Part 2 of this project, and any available conclusions will be reported in the 31 May 1999 report.

6 An "exact" computation for submerged bodies

Relatively recently there have become available a number of "exact" numerical procedures for solving free-surface potential flows about bodies of arbitrary shape. For surface ships, these include codes by Raven, Larsson, etc. Most of these codes are commercial in character, expensive to purchase and very expensive in computer time and effort to run, being suitable only for use on the fastest available supercomputers.

One of us [9] has developed a very efficient code for submerged bodies of arbitrary shape. This has already been shown to be accurate for spheres and bluff (1:5) spheroids, at submergences that can be reduced until the surface waves are about to break. This Scullen code is now available for testing on more elongated submerged bodies as part of the present project. This is intended to allow comparison to be made with the thin-ship code during Part 2. Such a comparison has advantages over other validation techniques in that it tests only the thin-ship approximation (replacing the body by a known source distribution on its centreplane and linearising the free surface), without confusion with other physical processes and approximations such as neglect of viscosity.

It is anticipated that comparisons of results produced from thin-ship theory with those produced by the more accurate nonlinear theory will be made for 1:8 and 1:10 spheroids, over a range of speeds and depths of submergence consistent with those for real submarine practice. This comparison will be both "internal" (comparison of the results of a full nonlinear run with those for the same Scullen program running in cut-down form, with the thin-ship approximations made internally) and "external", where the comparison is between full Scullen computations and the specific Michell integrals described earlier in the present report.

Results of such comparisons will be included in the 31 May 1999 report. Preliminary computations made already (at 30 April 1999) have already been performed indicating promising internal agreement. One point of note is that the computation time seems likely to be a factor of at least 5000 greater for the nonlinear Scullen code compared to that for evaluation of the Michell integrals on the same machine.

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