SEA WAVE PATTERN EVALUATION

Part 5 Report: Speed-up and Squat

E.O. Tuck, D.C. Scullen and L. Lazauskas

Scullen & Tuck Pty Ltd
PO Box 237 Rundle Mall, Adelaide, South Australia 5000

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Abstract

This is the fifth report in a series describing a computer program `SWPE` which provides the wave elevation and flow field created by a given surface-piercing or submerged body moving steadily forward in still water of constant depth, at all points, whether near to or far from the body.

The current release of the program, `SWPE5.0`, extends `SWPE4.0` in a number of directions, including increased accuracy and speed, automatic raising or lowering and rotating of the input hull, and computation of sinkage and trim.

The picture on the front cover of this report is a contour plot constructed from output from `SWPE`, for a Los Angeles Class submarine travelling at 10 knots when slightly more than half of its main body is submerged. The new automatic hull-raising feature of `SWPE5.0` is exercised in order to raise the vessel 10m relative to the (just-barely submerged to top of sail) position assumed by the input offsets. The red outline is an estimate of the non-submerged portion of the upper surface of the hull, based on the computed water surface; splashing and white water will change this outline in practice.

Introduction

The improvements made to SWPE under the current contract can be divided into two broad categories. The first are enhancements to the speed and accuracy of the program, and the second are additions to its functionality allowing the computation of sinkage and trim.

Within the first category, three tasks have been completed. Most important of these, a very considerable speed-up of the most computationally expensive component, namely the far field for points between bow and stern, has been implemented. The manner in which this has been achieved was described in the Appendix of the interim report [18]. Briefly, the speedup is via employment between bow and stern of a rapid method that had previously only been available aft of the stern of the ship. The result is that an entire row of points between bow and stern can now be calculated for not much more cost than that of a single point, with only a single integration required for each additional point. This is in stark contrast to the convolution method previously implemented in SWPE4.0, which required a separate triple integration at every field point. This component can now be calculated with a dramatic increase in speed and, depending upon the number of points within a row, increases in speeds by a factor of 50 can be achieved. Thus, this far-field component is no longer the slowest, with the near-field component now being the chief bottleneck.

To address this, an option has been introduced which eliminates the computation of the near-field component when its contribution is negligible compared with the local wave amplitude. It has been found that, over a range of practical operating speeds for a sample set of vessels, a rectangular region of length twice that of the vessel, and width equal to the vessel's length, satisfies this criteria. Thus, a simple switch has been introduced to SWPE5.0 so that beyond this region the local-field component is not calculated, and the total field is assumed to be identical to the far field.

The last of the enhancements to speed and accuracy is the implementation of a consistent quadratic-interpolation assumption of hull offsets. Previously, SWPE4.0 used a mixture of linear and quadratic assumptions when integrating near and far-field components over the hull. The new approach allows a more accurate solution with less computational effort.

The second category of tasks were those necessary for computation of sinkage and trim of the vessel. SWPE required modification to allow the evaluation of near-field velocity components both on the surface and within the fluid. This then allows evaluation of the pressure along the centreplane of the hull, and its integration to yield the hydrodynamic force and moment acting on the hull. Using these outputs, `SWPE5.0` now evaluates the sinkage and trim displacements of the hull necessary to balance the forces and moments so that the hull is in equilibrium.

In addition, the code has been enhanced so that the hull can be automatically raised/lowered and rotated by amounts given by the computed values of sinkage and trim, before the wave elevations are evaluated. To do this, it was necessary to introduce the ability to raise the vessel in the water by a specified amount relative to the notional "at dock" position in which its offsets are provided. This "negative submergence" is in addition to a positive submergence option which was already implemented in `SWPE4.0`. Implementation of the negative submergence option requires `SWPE` to automatically discard some given offsets, namely those that are now above the mean waterline, and to quadratically interpolate to find offsets at the new waterline. This option can be used explicitly, for example, to determine the free surface produced by a partially exposed submarine. A similar technique, also using quadratic interpolation of the offsets at the new waterline, was required to allow rotation of the vessel through small angles of trim.

Since this is the first occasion in which sinkage and trim, or squat, has been considered in the series of `SWPE` reports, we conclude this report with a detailed technical appendix indicating the nature of the squat computation problem, its implementation in `SWPE`, and the results compared to experiments.

Appendix: Squat

Introduction

The hydrodynamic pressure exerted on a ship's hull due to its forward motion causes forces which are capable of altering the attitude of the ship, inducing both a vertical movement or "sinkage" (positive downward) and a rotation or "trim" angle (positive when bow up). The combination of these two quantities is referred to as "squat".

Squat is small for displacement vessels, involving vertical displacements of only at most a few percent of the vessel's draft at most speeds. Sinkage is generally positive at normal speeds, i.e. the ship's effective draft is increased. This positive sinkage reaches a maximum at Froude numbers of the order of 0.5, then reduces and may go negative at very high speeds, although the latter phenomenon has seldom been investigated for displacement vessels.

However, at high speeds there is a connection between the squat phenomenon and the mechanism of planing, as a negative sinkage or rise in the water is induced by positive hydrodynamic lift. In that case however, for vessels capable of planing, the magnitude of squat is no longer small relative to the draft, and an alternative theory is required, e.g. one where the draft rather than the beam is assumed small relative to the shiplength.

We have programmed into the `SWPE` code a consistent thin-ship estimate of squat. This means that we integrate the linearised pressure on the centreplane to give approximations to the force and moment, and find the resulting sinkage and trim directly by solving hydrostatic equilibrium equations. For a thin ship, squat is formally of second order in the thin beam, so is small compared to both beam and draft of the ship; errors in computing squat due to this thin-ship approximation are formally even smaller, of third order in the beam.

In the following sections, the mathematical derivation of this theory is given, followed by indications of how it is implemented in the present version `SWPE5.0` of the code, and finally comparisons between results from this code and published model experiment data are made. The conclusion is generally favourable, with good prediction of qualitative trends, and quantitative accuracy in about the 20% error range, similar to that of other published theories. Possible improvements in the theory are discussed and may be implemented in a future version of `SWPE`.

Mathematical formulation

Under the assumptions of thin-ship theory, the net upward force acting on the ship's hull due to fluid pressure is

$$F = 2 \iint_B p(x, 0, z) Y_z(x, z) dx dz \tag{1}$$

where

$$p(x, y, z) = -\rho g z - \rho U \Phi_x \tag{2}$$

is the excess pressure over atmospheric, the first term being the hydrostatic pressure and the second the hydrodynamic pressure. In the hydrodynamic pressure,

$$\Phi(x, y, z) = 2U \iint Y_\xi(\xi, \zeta) G(x - \xi, y, z; \zeta) d\xi d\zeta \tag{3}$$

is the disturbance velocity potential. Similarly, the stern-up moment about the origin due to the fluid pressure is

$$M = 2 \iint_B x p(x, 0, z) Y_z(x, z) dx dz. \tag{4}$$

For later reference, we write $F = F_1 + F_2$ and $M = M_1 + M_2$ where subscript 1 denotes hydrostatic contributions, and the hydrodynamic parts of the force and moment are

$$F_2 = -4\rho U^2 \iint dx dz \iint d\xi d\zeta Y_x(\xi, \zeta) Y_z(x, z) G_x(x - \xi, 0_\pm, z; \zeta) \tag{5}$$

and

$$M_2 = -4\rho U^2 \iint x dx dz \iint d\xi d\zeta Y_x(\xi, \zeta) Y_z(x, z) G_x(x - \xi, 0_\pm, z; \zeta) \tag{6}$$

It is presumed that the given offsets describe the vessel when in a state of static equilibrium, in which the total force and moment (due to both fluid pressure and mass distribution) balance when the ship is at rest. If we write $F_1 = F_0$ and $M_1 = M_0$ for the hydrostatic force and moment corresponding to these given offsets, then this means that F_0 is equal to the (fixed) weight of the vessel, and $x = M_0/F_0$ is its (fixed) centre of buoyancy. Then when the vessel is in motion at speed U, we must have $F = F_0$ and $M = M_0$, the integrals for F and M being carried out over the actual dynamic hull surface.

If we assume that when the ship is in motion, there is a small sinkage σ (positive σ means an increased draft) and a small bow-up trim angle τ

radians, then the section at station x is lowered in the water by $\sigma + x\tau$, so the effective submerged cross-sectional area is increased by

$$\Delta S(x) = (\sigma + x\tau)B(x) \tag{7}$$

where $B(x)$ is the local waterline width. The change $\Delta F = F_1 - F_0$ in the upward hydrostatic force due to such sinkage and trim is therefore

$$\begin{aligned} \Delta F &= \rho g \int \Delta S(x) dx & (8) \\ &= \rho g \left(\sigma A_W + \tau x_F A_W\right) & (9) \end{aligned}$$

where $A_W = \int B(x)dx$ is the waterplane area and $x = x_F = \int xB(x)dx/A_W$ is the centre of flotation. Similarly, the change $\Delta M = M_1 - M_0$ in the stern-up hydrostatic moment due to sinkage and trim is

$$\begin{aligned} \Delta M &= \rho g \int x \Delta S(x) dx & (10) \\ &= \rho g \left(\sigma x_F A_W + \tau I_W\right) & (11) \end{aligned}$$

where $I_W = \int x^2 B(x)dx$ is the moment of inertia of the waterplane. On the other hand, to leading order the integrals in (5) and (6) for the hydrodynamic force F_2 and moment M_2 can be carried out over the original static hull surface.

Now for equilibrium we must have $F_0 = F = F_1 + F_2$ and $M_0 = M = M_1 + M_2$, i.e. $\Delta F = -F_2$ and $\Delta M = -M_2$. Solving these equations simultaneously yields expressions for the sinkage σ and trim angle τ, namely

$$\sigma = -\frac{F_2}{\rho g A_W} - \tau x_F \tag{12}$$

where

$$\tau = \frac{-M_2 + F_2 x_F}{\rho g (I_W - A_W x_F^2)}. \tag{13}$$

Computational method

The present version, `SWPE5.0` of the computer code assumes initially that the hull is in an attitude specified in the input file by user values of sinkage and trim, and takes action to adjust these values depending on the value of a squat iteration parameter. This parameter is an input integer, presently allowed to take the values 0 and 1 only.

If the squat iteration parameter is set to zero, and the specified sinkage and trim are zero, the behaviour of SWPE5.0 is identical to that of SWPE4.0, in that no action is taken to adjust input offsets, and no sinkage and trim is computed or used. If the squat iteration parameter is zero but the user specifies non-zero values for the sinkage and trim, the program will use those values, adjusting the input offsets appropriately before performing any flow computations, but will not calculate hydrodynamic forces on the hull nor in any way change the specified sinkage and trim.

On the other hand, if the user specifies in the input file that the number of squat iterations is equal to 1, then (after adjusting the input offsets to take account of the specified sinkage and trim if any) SWPE5.0 will calculate the (out-of-balance) upward hydrodynamic force F_2 and stern-up hydrodynamic trimming moment M_2 using equations (5) and (6). The new sinkage σ and trim τ due to F_2 and M_2 are then calculated using equations (12) and (13). Finally, the hull offsets are rotated and translated by τ and σ, respectively, before any further flow calculations (such as wave elevations or hull wave profiles) are performed.

The above procedure thus implements only a consistent thin-ship approximation to the sinkage and trim. Although we shall show below some reasonable comparisons of squat results with experiment (mostly within 20% error bounds) using this approximation only, there are clearly improvements that could be made. These include viscous effects, pressure integration over the actual hull rather than the centreplane, inclusion of other velocity components, and iteration of input hull data until net forces and moments are zero. In a future implementation of SWPE, the squat iteration parameter may be allowed to take values greater than 1, in which case the above-described process will repeat as many times as this parameter specifies.

Inclusion of the effect of viscous forces and moments on trim and sinkage requires calculation of the longitudinal distribution of the frictional forces on the wetted hull, which in turn requires calculation of the wetted surface area, and the centroid of the wetted surface [2]. Although the present version SWPE5.0 does not yet deliver these capabilities, the computer code contains several routines and functions that will assist in their ultimate implementation. Thus, for example, SWPE5.0 allows calculation of the hull wave profile, which is useful not only in its own right, but can also be used in an accurate calculation of the wetted surface area. Also, the longitudinal centre of buoyancy (LCB) and vertical centre of buoyancy (VCB) are calculated for the hull in its static and squatted attitudes.

In SWPE5.0 only the x component of velocity is used in the calculation of F_σ and M_τ. This is consistent with the assumptions of thin-ship theory. However, squat predictions might be improved by using all three velocity components in the calculation of the pressure $p(x, y, z)$ by Bernoulli's equation, and also by integration of the resulting pressure $p(x, \pm Y(x, z), z)$ over the actual hull surface $y = \pm Y(x, z)$ rather than the centreplane $y = 0_\pm$.

These enhancements are recommended for inclusion in future versions of SWPE. In the meantime the simplest SWPE5.0 squat formulation is tested in the following sections by comparison with published experimental data.

Comparisons with experiments and other prediction methods

We now compare SWPE5.0 predictions of sinkage and trim with experimental data and with the results of other computer models. We also examine the effects of the inclusion of squat on wave elevations and hull wave profiles for some hulls. In the results to follow, sinkage is presented in the non-dimensional form $1000\sigma/L$. Trim is given in degrees.

Table 1 summarises the geometric parameters of the hulls used in our comparisons. In the table, L is the length of the hull in metres, D is the displacement volume, B is the beam, T is the draft, x_F is the longitudinal centre of flotation (LCF), x_G is the longitudinal centre of gravity (LCG), and C_P is the prismatic coefficient. The ratio $L/D^{1/3}$ is sometimes referred to as the "slenderness" coefficient or as the "fineness" coefficient, and should be large for thin ships.

Wigley parabolic hull

The Wigley parabolic hull is a standard test for many hydrodynamic codes and predictions. With a length-to-beam ratio of 10, the hull can be considered to be a "thin ship". However, reference to Table 1 shows that the slenderness coefficient is 7.114, which is among the smallest for the hulls considered in the present report.

Hull	L (metres)	$L/D^{1/3}$	L/B	B/T	x_F/L	x_G/L	C_P
Wigley	1.800	7.114	10.000	1.600	0.000	0.000	0.667
Lego 7	1.875	7.617	12.500	1.600	0.054	0.054	0.850
Lego 8	2.063	8.187	13.750	1.600	0.033	0.033	0.828
Lego 11	2.625	9.371	17.500	1.600	0.040	0.040	0.893
Lego 12	2.813	9.890	18.750	1.600	0.025	0.025	0.874
NPL 3B	1.600	6.270	7.000	2.000	0.084	0.063	0.693
NPL 6A	1.600	9.503	15.100	1.500	0.084	0.063	0.693
DTMB 5415	5.720	6.986	7.442	2.053	0.047	0.007	0.630

Table 1: Geometric parameters for the hulls used in comparisons with experiments and with other theoretical models.

Sinkage and trim

Figures 1 and 2 show `SWPE5.0` predictions for the sinkage and trim of the Wigley hull as a function of Froude number.

Experimental results for sinkage and trim were reported in [5]. Also shown in the graph are `SWPE5.0` predictions using three different panel densities (that is, the number of stations and waterlines used to represent the hull), namely 11×11, 21×21, and 41×41. Predictions for the three panel densities are very similar to each other, which is a consequence of the consistent quadratic interpolation of hull offsets that `SWPE5.0` now makes, and even the coarsest 11×11 data seem adequate. Doctors and Day [2] found that a panel layout of 41 sections and 8 waterlines gave sufficient accuracy in their numerical experiments.

It can be seen in Figure 1 that `SWPE5.0` under-predicts the sinkage in the range of Froude numbers considered here, with the greatest errors for large Froude numbers, where even qualitative agreement is lost for $F > 0.65$. However, the agreement at lower Froude numbers is reasonable, with less than 20% error in the middle range of Froude numbers, say $0.3 < F < 0.55$, where the sinkage is greatest.

`SWPE5.0` estimates for the trim of the Wigley hull are in close agreement for Froude numbers below $F = 0.45$; for $F > 0.45$, `SWPE5.0` underpredicts the trim by about 20%.

Hull wave profile

Figure 3 shows the wave profile along a Wigley hull for Froude number 0.316. Experimental results were taken from [11], as were the predictions of the non-linear code RAPID, its linear predecessor Dawson, and a Neumann-Kelvin method. The curve labelled "SWPE: Static" corresponds to the case where the hull is in its unsquatted attitude. The curve labelled "SWPE: Squatted" is for the hull in the attitude predicted by `SWPE5.0`.

The differences between the two `SWPE5.0` predictions are quite small, as is to be expected because the sinkage and trim are small for this Froude number. The source of discrepancy between theory and experiment is not to be sought in sinkage and trim, at least at this speed. Agreement is good qualitatively, and moderate quantitatively, and `SWPE5.0` performs about as well as the other codes.

There are small oscillations in the `SWPE5.0` curves which correspond to the diverging waves shed from the hull. These are not seen in the experiments or the other codes, which suggests that viscous damping of these short waves may play a role.

Doctors and Day's "Lego" hulls

The hulls in this section are part of a systematic series described in detail in [2]. Essentially, the hulls in this series have the same bow section and stern sections as the standard Wigley parabolic hull, but a length of parallel middlebody has been inserted between the bow and stern sections; the stern end is also truncated to create a transom.

Although Doctors and Day present both sinkage and trim results, there is some uncertainty about the origin of co-ordinates used both for the experiments and the Doctors and Day theory. This uncertainty affects sinkage only; hence we provide comparisons here only for trim. Experimental results and predictions by Doctors and Day were taken from hand-faired curves of enlarged graphs in [2], and are shown in Figures 4 to 7.

The two prediction methods both capture well the general trend of the trim curve. For the Lego 7 model (Figure 4), Doctors and Day over-predict the trim for all Froude numbers and `SWPE5.0` under-predicts it, both with errors of the order of 20%. At low Froude numbers there are oscillations, which

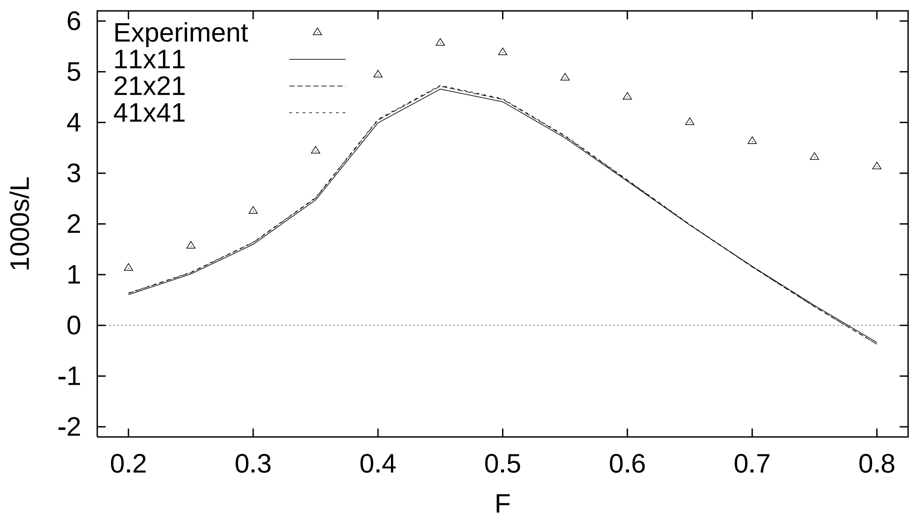

Figure 1: Sinkage of a Wigley hull.

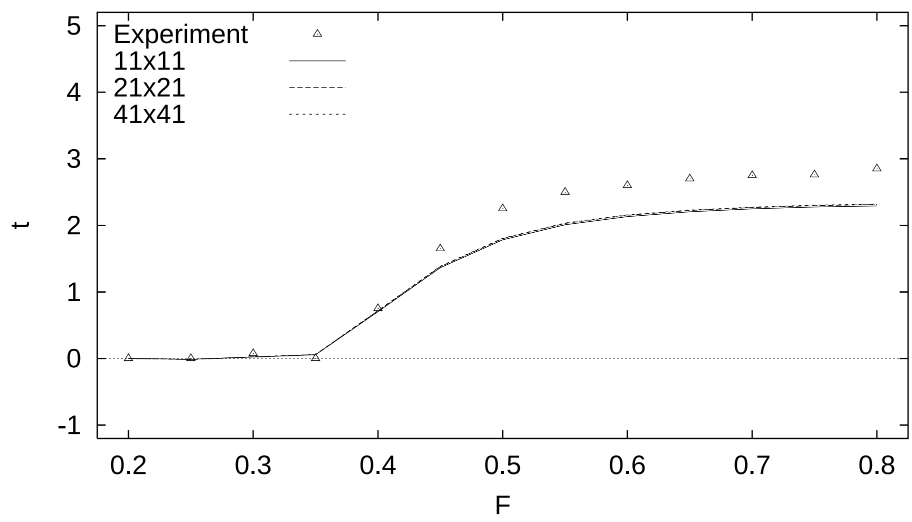

Figure 2: Trim of a Wigley hull.

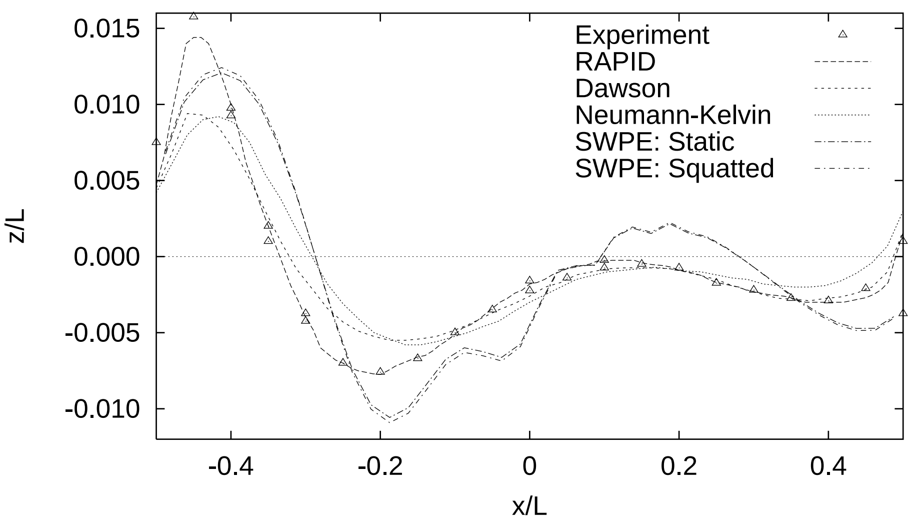

Figure 3: Experimental and predicted wave elevation profiles on a Wigley hull at Froude number 0.316.

have also been noted by Yeung ([19] p. 52), who commented that oscillations in trim curves are generally larger for hulls with small length-to-beam ratio, and that hulls with larger block coefficients display large oscillations.

For the Lego 8 model, `SWPE5.0` is a somewhat better estimator of trim (Figure 5) than Doctors and Day, with errors of the order of 10% compared to Doctors and Day's over-estimate of about 30%.

For the Lego 11 hull, trim predictions (Figure 6) are also reasonable, as `SWPE5.0` under-estimates the larger trim values by 20-25%.

Finally for the most slender Lego 12 hull, the two theories both agree well with experiment for trim (Figure 7).

NPL Series hulls

The NPL series of hulls is representative of modern high-speed displacement vessels. They have a round bilge, a transom stern, and bow sections that are flared near the design waterline. The geometric particulars are summarised in Table 1. Experimental results were taken from tables in the report by Molland, Wellicome and Couser ([9]); an earlier report by Bailey ([1]) concerning the same hull series also provides valuable information. The NPL3b hull has

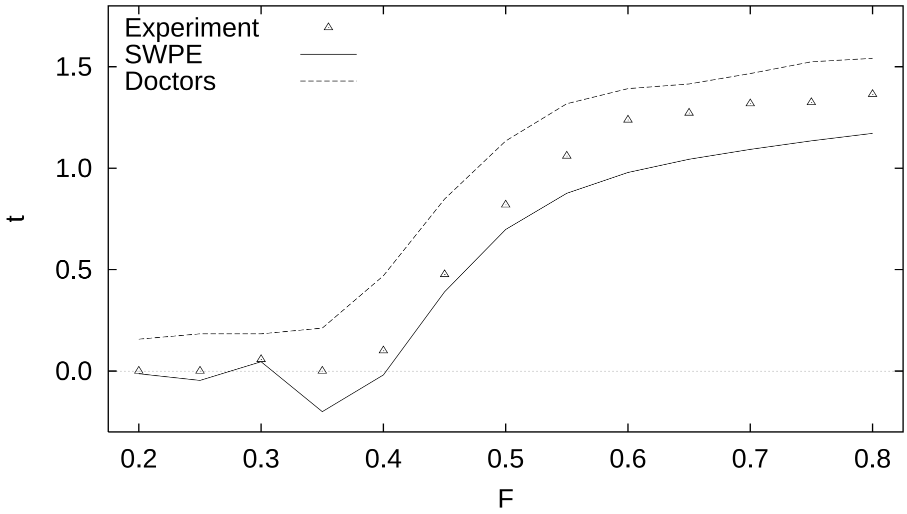

Figure 4: Trim of a Lego 7 hull.

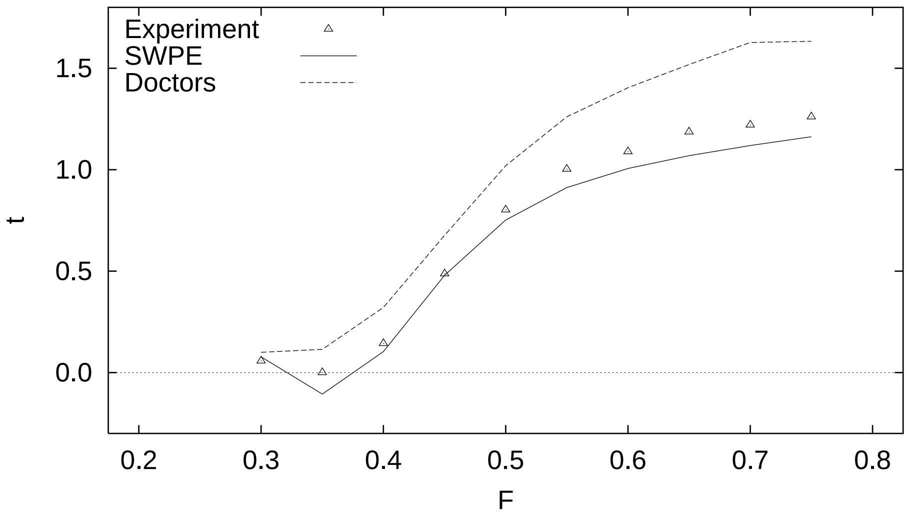

Figure 5: Trim of a Lego 8 hull.

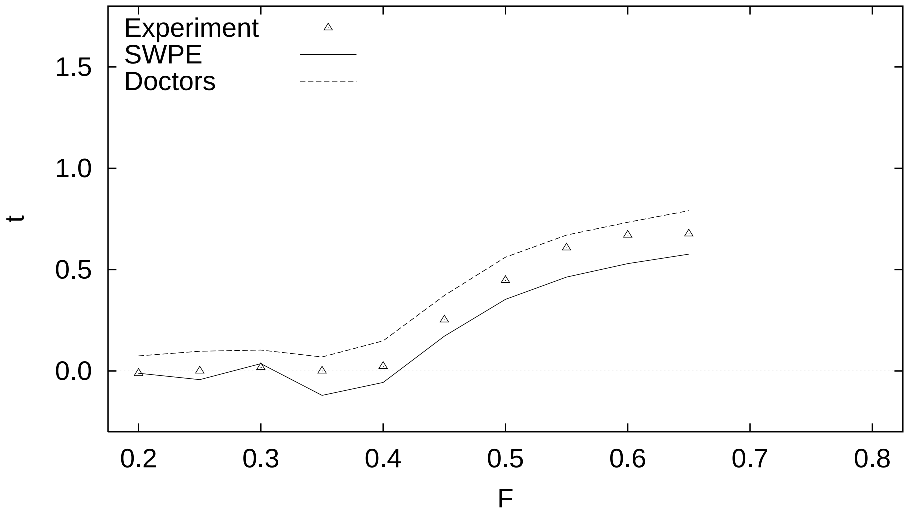

Figure 6: Trim of a Lego 11 hull.

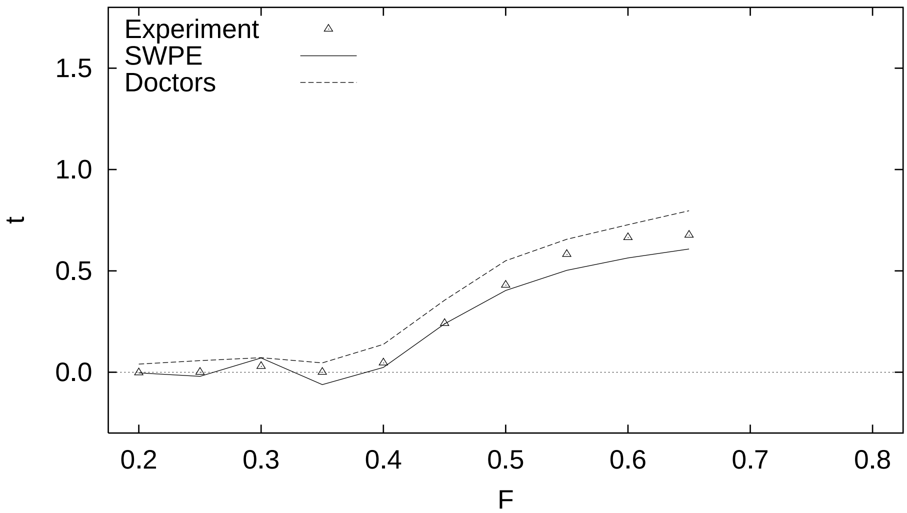

Figure 7: Trim of a Lego 12 hull.

F	Exp. 1 σ (m)	Exp. 2 σ (m)	SWPE σ (m)	Exp. 1 τ (deg.)	Exp. 2 τ (deg.)	SWPE τ (deg.)
0.2755	0.009	0.010	0.009	-0.063	-0.105	0.034
0.4136	0.020	0.025	0.025	0.445	0.445	1.009

Table 2: Comparison of experimental sinkage and trim with `SWPE5.0` predictions for the 5415 hull at two Froude numbers. Sinkage is at midships; trim is in degrees.

the smallest length-to-beam ratio of the hulls examined in [9]; conversely NPL6a is the finest hull in that series.

Sinkage and trim

`SWPE5.0` under-predicts the sinkage of the NPL3b hull for low Froude numbers (Figure 8); at higher Froude numbers, the sinkage is surprisingly well predicted. The sinkage of the NPL6a hull (Figure 10) is under-predicted for all Froude numbers.

The errors for NBL3b are less than those for NBL6a, being within about 20% for the former in the range of significant sinkage, but about 25-30% for the latter. In addition, the experimental results for the NPL6a hull show a local minimum at about $F = 0.65$ followed by increased sinkage at higher speeds, a feature which is typical of the finer representatives of this hull series ([9] pp. 37–38), and seems not to be captured by `SWPE5.0`.

`SWPE5.0` under-predicts the trim of the NPL3b hull significantly for all Froude numbers (Figure 9), whereas its trim prediction for the NPL6a hull is excellent (Figure 11).

DTMB hull 5415

The geometric particulars for this destroyer hull are summarised in Table 1. This hull was also used as an example in previous `SWPE` reports ([16], [17]).

Table 2 compares `SWPE5.0` predictions of sinkage and trim with two sets of experiments. The first set of experimental results are those obtained during the "Wake-off" ([6]). The second set of experimental results were determined

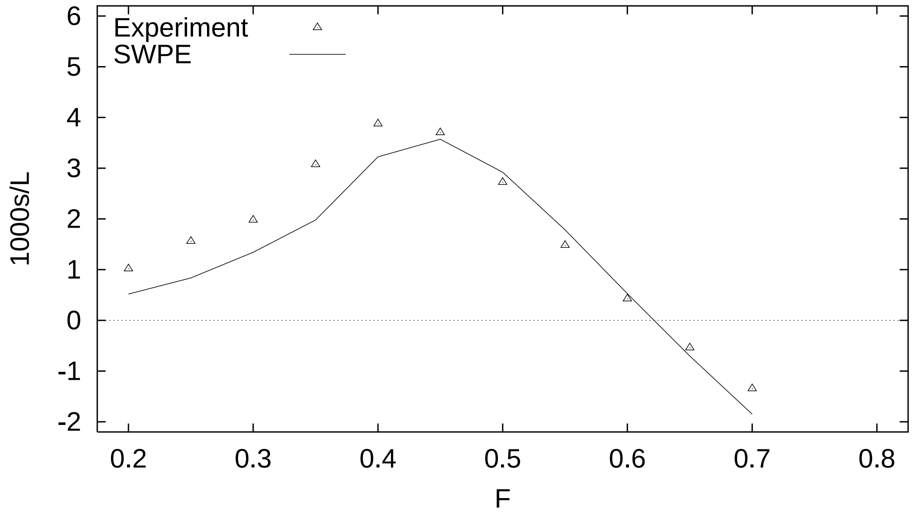

Figure 8: Sinkage of an NPL3b hull.

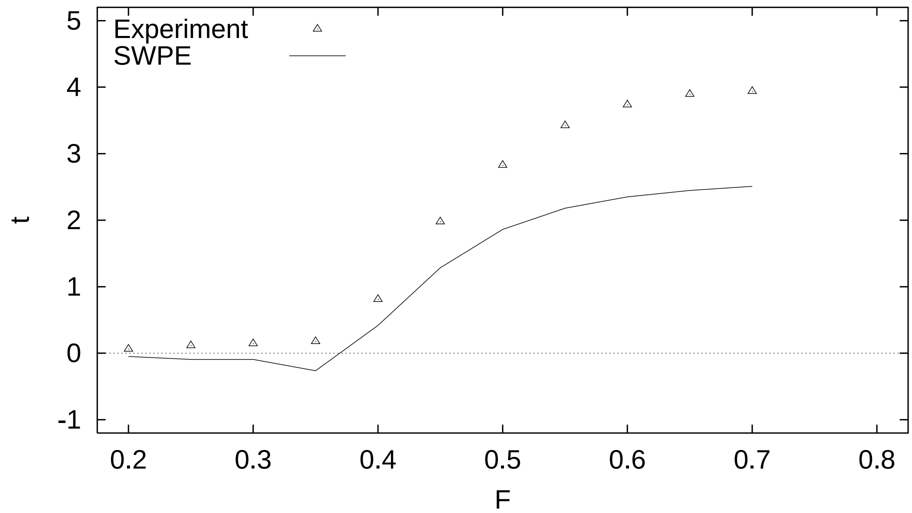

Figure 9: Trim of an NPL3b hull.

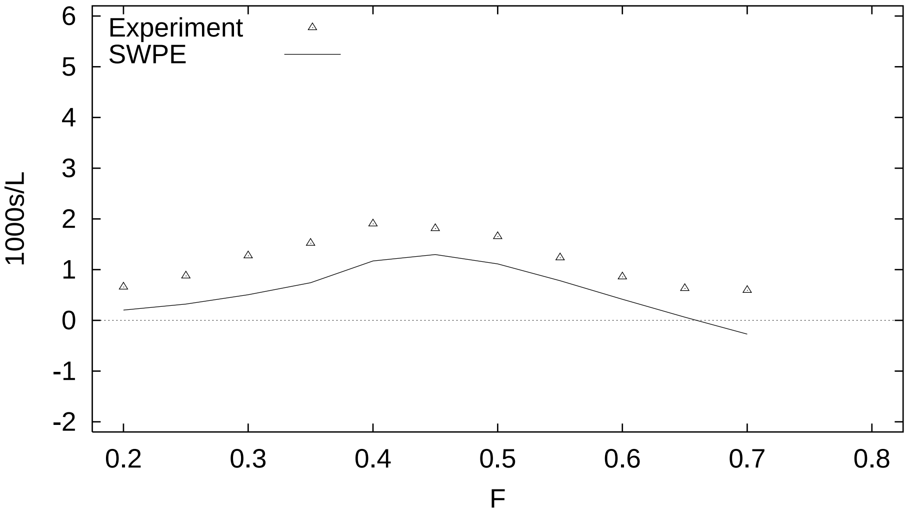

Figure 10: Sinkage of an NPL6a hull.

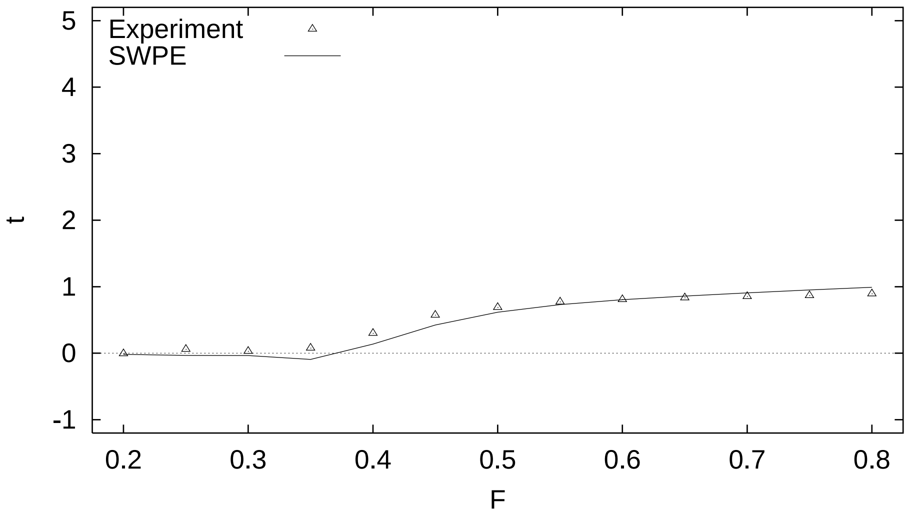

Figure 11: Trim of an NPL6a hull.

during resistance measurements on the same hull, but at a later date (see the DTMB WWW site for further details). In the comparisons, `SWPE5.0` does very well as a predictor of sinkage for both Froude numbers; trim is less well predicted.

Wave cuts

The effect of sinkage and trim on wave elevations is shown in Figures 12 and 13, where the wave elevation and the x-ordinate have been scaled by $k = g/U^2$.

Four curves are given in each figure. Experimental results were taken from the "Wake-Off" report by Lindenmuth et al ([6]). The curve labelled "SWPE: Static" shows `SWPE5.0` predictions for the hull in its unsquatted attitude. The curve labelled "SWPE: DTMB" shows `SWPE5.0` predictions when the hull is in the attitude measured during the experiments (the values in the columns labelled "Exp. 1" in Table 2). The curve labelled "SWPE: Squatted" shows predictions for the hull in the attitude predicted by `SWPE5.0`.

As with the Wigley results in Figure 3, it seems unlikely that squat is a major determining factor in discrepancies between theory and experiment for wave profiles. The differences between `SWPE5.0` computations with and without inclusion of squat are in general small compared to the differences between any of the `SWPE5.0` results and experiment.

References

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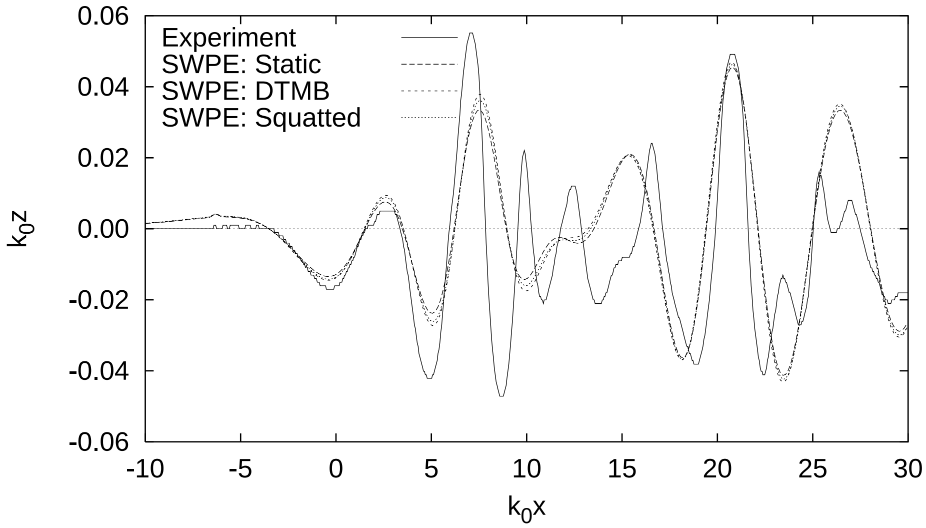

Figure 12: Effect of squat on the wave elevations at $y = -1.854$m from the ship's track produced by `SWPE` for the 5415 hull at a Froude number of 0.2755.

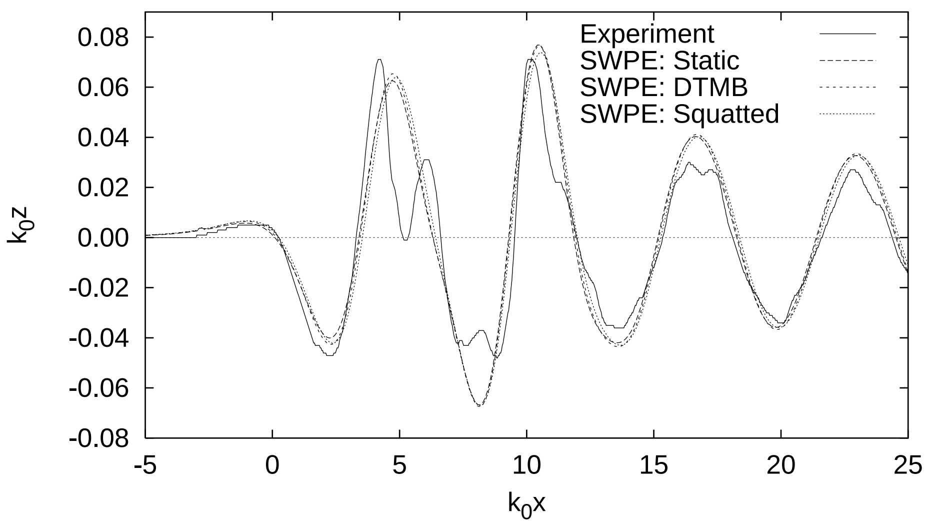

Figure 13: As above but with Froude number 0.4136.

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