SEA WAVE PATTERN EVALUATION

Part 6 Report: Viscosity Factors

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Abstract

This is the 6th report in a series describing a computer program `SWPE` which provides the wave elevation and flow field created by a given surface-piercing or submerged body moving steadily forward in still water.

The present report discusses implementation of a correction factor in `SWPE` to account for viscous dissipation of the waves.

Introduction

In SWPE3.0 and all versions following that, an option was presented to allow a correction for viscous damping of the far-field waves computed by SWPE. At the outset it must be stated that if one uses the usual molecular kinematic viscosity for water, of the order of 10^{-6} m^{2}s^{-1}, there is no discernible effect. Only with viscosities several orders of magnitude larger are converged results from SWPE different from those with zero viscosity.

Even with large viscosities of the order of 10^{-3} m^{2}s^{-1}, as originally recommended for SWPE3.0, the viscous effect seems to be quite small. Such large viscosities might be possible (though we believe unlikely) as ambient eddy viscosities in the ocean. However, we are more interested in eddy viscosity effects produced by the vessel itself, especially in its wake, where the highly vortical flow might indeed correspond to effective eddy viscosities of up to that order of magnitude, although we now recommend somewhat lower viscosities.

An incidental benefit of introduction of viscosity is an improvement in the achieved accuracy (per unit of computational effort) of SWPE. This occurs because in the far field the most computationally intense work is devoted to capturing the extreme diverging waves, travelling nearly perpendicularly to the ship's track. These are the shortest-wavelength, highest-frequency and slowest-moving waves in the spectrum, and hence are very strongly and preferentially damped by the viscous correction. Hence even when the viscosity is too small to produce a visible effect on the final wave pattern, its inclusion enables that pattern to be computed more accurately. This is most important if SWPE is to be used for bluff bodies moving at high speeds; for conventional bodies moving at conventional speeds, SWPE has ample accuracy even with zero viscosity, providing that parameters controlling its accuracy are set to the recommended values.

The empirical viscous correction factor implemented in SWPE3.0 was derived from a formula for damping of plane water waves given by Lamb (1932), p. 625. Lamb suggested that a plane wave of wavenumber k would be damped with respect to time t as it travelled, the damping factor being

$$V = \exp\left(-2\nu k^2 t\right) \tag{1}$$

where ν is the kinematic viscosity. In the far field of a ship moving at steady speed U in the $-x$ direction, we have a collection of such plane waves travelling at angles θ to the ship's track, with wave number $k = k_0 \sec^2 \theta$

where $k_0 = g/U^2$, so the damping factor to be applied to waves propagating at angle θ would be

$$V = \exp\left(-2\nu k_0^2 t \sec^4\theta\right). \tag{2}$$

A naive translation of the Lamb factor to steady flow is to replace t by x/U, to give

$$V = \exp\left(-2\nu k_0^2 \frac{x}{U} \sec^4\theta\right) . \tag{3}$$

This is the assumption made in `SWPE3.0`, which was never intended as other than a first-cut empirical estimate of the effect of viscosity, and can be challenged on a number of grounds. In particular, it seems rather arbitrary that it depends only on x, and not at all on the observing distance y from the centreline. Secondly, it uses a constant value for the eddy viscosity ν, whereas in the present context we are interested in viscosities that are in some way concentrated in the wake. In particular, ν may decrease as y increases, and then the history of the wave's location becomes important. Also, at least for the two-dimensional case described by Lamb, there are strong grounds to suggest that group velocity should be used rather than phase velocity, and then t would be replaced by $2x/U$ (Zilman 2001) rather than x/U, resulting in a change of the constant coefficient from 2 to 4 in equation (3). That said, Zilman also arrives at the original `SWPE3.0` formula when he considers the case of strongly divergent waves.

In the present report we investigate some of these matters, implementing various alternative plausible factors and testing the extent to which `SWPE` results change.

Non-uniform viscosity

At first, one might think that replacing ν = constant by a non-uniform $\nu = \nu(x, y)$ within the decay factor formula would yield a meaningful result. For example, one could perhaps generalise the `SWPE3.0` formulation to

$$V = \exp\left(-2\nu(x, y) k_0^2 \frac{x}{U} \sec^4\theta\right) . \tag{4}$$

However, this would be incorrect, since this factor retains no information about the history of the wave's propagation. Thus, far away from the ship where $\nu(x, y)$ could be expected to be small (essentially zero), there would then be unattenuated high-frequency waves. This of course cannot be true, since waves are in fact generated by the ship and all short-wavelength waves are rapidly diminished by the high viscosity there, leaving none to propagate to the outer regions!

The correct generalisation to non-uniform viscosity is to account for the history of the wave's propagation by replacing νt in equation (2) by its time-dependent equivalent, yielding

$$V = \exp\left(-2k_0^2 \sec^4\theta \int_0^t \nu(\tau)d\tau\right). \tag{5}$$

What remains to be done is to relate this elapsed time t to some spatial characteristic, and to specify the viscosity ν as a function of position, and hence of elapsed time. (For example, in `SWPE3.0`, $\nu =$ constant and $t = x/U$.)

Further, it is our belief that if viscosity within the fluid depends upon spatial location, then it depends more strongly on y than it does on x. This is in contrast to the actual decay factor, which we believe depends more strongly on x than y. If we assume that viscosity is independent of x, i.e. $\nu = \nu(y)$, and $dy/dt \neq 0$, then we can transform the integral in the above to give

$$V = \exp\left(-2k_0^2 \sec^4\theta \int_0^{y(t)} \nu(\eta)\frac{d\tau}{d\eta}d\eta\right). \tag{6}$$

Alternative viscosity models

A possible choice for t is the time taken for a wave travelling in the θ-direction to propagate from the centreline to the observation point (x, y). Since steady-state waves have speed $U\cos\theta$, the magnitude of their velocity component perpendicular to the ship's track is $U\cos\theta\sin|\theta|$, and $t_1 = \sec\theta \operatorname{cosec}|\theta||y|/U$. (Note that $y > 0$ for waves propagating with $\theta < 0$, and $y < 0$ for $\theta > 0$.) Thus $|dt/dy| = \sec\theta \operatorname{cosec}|\theta|/U$, and

$$V = \exp\left(-2\frac{k_0^2}{U}\sec^5\theta \operatorname{cosec}|\theta| \int_0^{|y|} \nu(\eta)d\eta\right). \tag{7}$$

If viscosity is assumed to be constant, this simplifies to

$$V_1 = \exp\left(-2\frac{k_0^2}{U}\nu|y|\sec^5\theta \operatorname{cosec}|\theta|\right). \tag{8}$$

An alternative for t is the time that has passed since the vessel was at the point from which these waves have propagated. Waves at (x, y) propagating in the direction θ can be traced back to their point of origin $(x+|y|\cot|\theta|, 0)$ on the centreline. The time elapsed since the vessel was at that point is $t_2 = (x+|y|\cot|\theta|)/U$. Note that this is not the same time as that required for the wave to propagate to its current location, as was given above, and therefore it is not appropriate to use equation (6). Instead, we must resort to equation (5) which, for constant viscosity yields

$$V_2 = \exp\left(-2\frac{k_0^2}{U}\nu\sec^4\theta\,(x+|y|\cot|\theta|)\right). \tag{9}$$

Another alternative is a combination of the above two, which exploits the difference in the two choices for t. That is, the amplitude of the waves is assumed to decay first due to constant viscosity for the period of time between when the vessel created the disturbance and when the wavefront began to propagate, and second due to the actual wave propagation as given by equation (7). This acknowledges the decay in amplitude of successive waves that radiate from the centreline. Thus

$$V = \exp\left[-2\frac{k_0^2}{U}\sec^5\theta\left((x\cos\theta - |y|\sin|\theta|)\nu(0) + \operatorname{cosec}|\theta|\int_0^{|y|}\nu(\eta)d\eta\right)\right]. \tag{10}$$

Note that the last term is the same as equation (7), and is that which is due to the propagation of the wave. The remaining terms are due to the time difference between the instant when the ship first created the disturbance and that when the wave started to propagate from the centreline. This is zero for the outermost wavefront, but nonzero for those that follow. If the viscosity is constant, then equation (10) simplifies to equation (9).

There are other possible alternatives, based on quite different ideas to those expounded above. For example, the Rayleigh-like derivation in Appendix 1 yields a combination $x\cos\theta + y\sin\theta$ of x and y equivalent to the phase of a wave propagating at angle θ, but with a multiplying factor $2\sec\theta$, namely

$$t_3 = 2\sec\theta(x\cos\theta + y\sin\theta)/U = 2(x+y\tan\theta)/U \tag{11}$$

for which

$$V_3 = \exp\left(-4\frac{k_0^2}{U}\nu\sec^4\theta\,(x+y\tan\theta)\right). \tag{12}$$

A case can also be made for an exponent half as large, i.e.

$$t_4 = \sec\theta(x\cos\theta + y\sin\theta)/U = (x+y\tan\theta)/U\ , \tag{13}$$

with

$$V_4 = \exp\left(-2\frac{k_0^2}{U}\nu \sec^4\theta\,(x + y\tan\theta)\right) \tag{14}$$

which, for waves propagating away from the centreline, is identical to the first term in equation (10) and thus represents the decay that occurs between the instant when the ship first creates the disturbance and that when the wave starts to propagate from the centreline.

Both cases V_1 and V_2 suffer from undesirable behaviour which unduly damps waves propagating in the direction of the ship (i.e. for small θ) due to the $\operatorname{cosec}\theta$ terms which they possess. V_1 is especially undesirable since it is independent of x. On the other hand, V_2 agrees with Zilman's (2001) formulation for strongly-divergent waves, i.e. for $\theta \to \pm\pi/2$. The options V_2 and V_4 each have the property that they reduce to the original option with $t_0 = x/U$ as programmed in `SWPE3.0` when we delete the dependence on y, and hence each gives the same decay with x of the waves along the track of the ship (which, as we noted previously, differs in the exponent by a factor of two from that which would be expected from arguments based on the group velocity). However, V_2 and V_4 have quite different behaviour for the new term depending on y. In fact, in spite of their apparently quite distinct form, there is a close relationship between V_2 and the (doubled viscosity) case V_3, associated with the principle of stationary phase, as discussed in Appendix 2, and these two options seem to decay the dominant far field waves identically. Importantly, the case V_3 reduces to $t = 2x/U$ for $\theta = 0$, consistent with the group-velocity formulation for two-dimensional waves, and we have decided that this is the best choice for future use in `SWPE`.

Computational results

Figure 1 shows a cross-sectional cut at $k_0x = 23$ of the waves made by a destroyer travelling at a Froude number of speed of 0.4136. The resulting plots of k_0Z versus k_0y shown are those for the viscous factor V_3, for viscosities ranging in decades from 10^{-5} to 10^{-2}. Curves for the factor V_4 would be the same as for V_3, but with twice the given value of viscosity.

These results are fully converged, i.e. the main accuracy parameter `Ntheta` of `SWPE` is increased until there is no change to graphical accuracy. This happens within the normal recommended range (say `Ntheta`< 4000), providing at the same time the recommended "fade factor" is used. This fade factor was included in `SWPE` originally mainly in order to provide a "pseudo-stationary-phase" character to the θ-integration in the extreme far field, but additionally has some of the characteristics of a small viscosity, of the order of 10^{-5}. If the viscosity is that small or smaller, and no fade factor is used, the required `Ntheta` for full convergence can be quite large, of the order of millions. Nevertheless, no fade factor was used for Figure 1, in order to enable unbiased estimates of the true effects of viscosity.

There is essentially no effect of viscosities smaller than 10^{-5}. The main residual effect of very small viscosity is seen very close to the track, i.e. for small y, where there are very short diverging waves, whose amplitude tends to zero as $y \to 0$, but at a more rapid rate as the viscosity is increased. As ν increases to 10^{-4} and higher, this damping of diverging waves spreads to higher and higher y values. By $\nu = 10^{-3}$ there are also reductions in the amplitudes of waves near the Kelvin angle (the highest crests and troughs), and then by $\nu = 10^{-2}$ the transverse waves (e.g. at $y = 0$) begin to be damped. It seems likely that the last property is undesirable, and perhaps even the reduction in the highest wave amplitudes should be avoided. A value of the order of $\nu = 10^{-4}$ seems a reasonable compromise, and is recommended for use in `SWPE6.0`.

We have also made similar runs with `SWPE3.0`, i.e. with the original x-dependent viscosity factor V. We have found that, providing the input viscosity to `SWPE3.0` is double that for `SWPE6.0`, consistent with setting $y = 0$ in the factor V_3, the results are almost graphically indistinguishable. There are about 4% differences with a viscosity $\nu = 10^{-3}$, which is higher than that now recommended, but less than 1% differences with the presently recommended $\nu = 10^{-4}$. Similarly, runs using V_4 produce almost identical results except that this factor has undesirable behaviour very near to $y = 0$, and is not recommended.

Our conclusion is that the formulation V_3 is the best option, since it has the same centreline behaviour as would be predicted from Lamb's two-dimensional formulation (using the group velocity rather than the phase velocity, which we now believe to be correct), and has the same outer-Kelvin-wake behaviour as also could be expected from both (a) the diverging wave formulation due to Zilman and Miloh (2001), and (b) the formulation V_2 based upon the time since the vessel created the wave (as is shown in Appendix 2). We therefore adopt this formulation for implementation in `SWPE6.0`.

However, we further conclude that the original empirical dissipation fac-

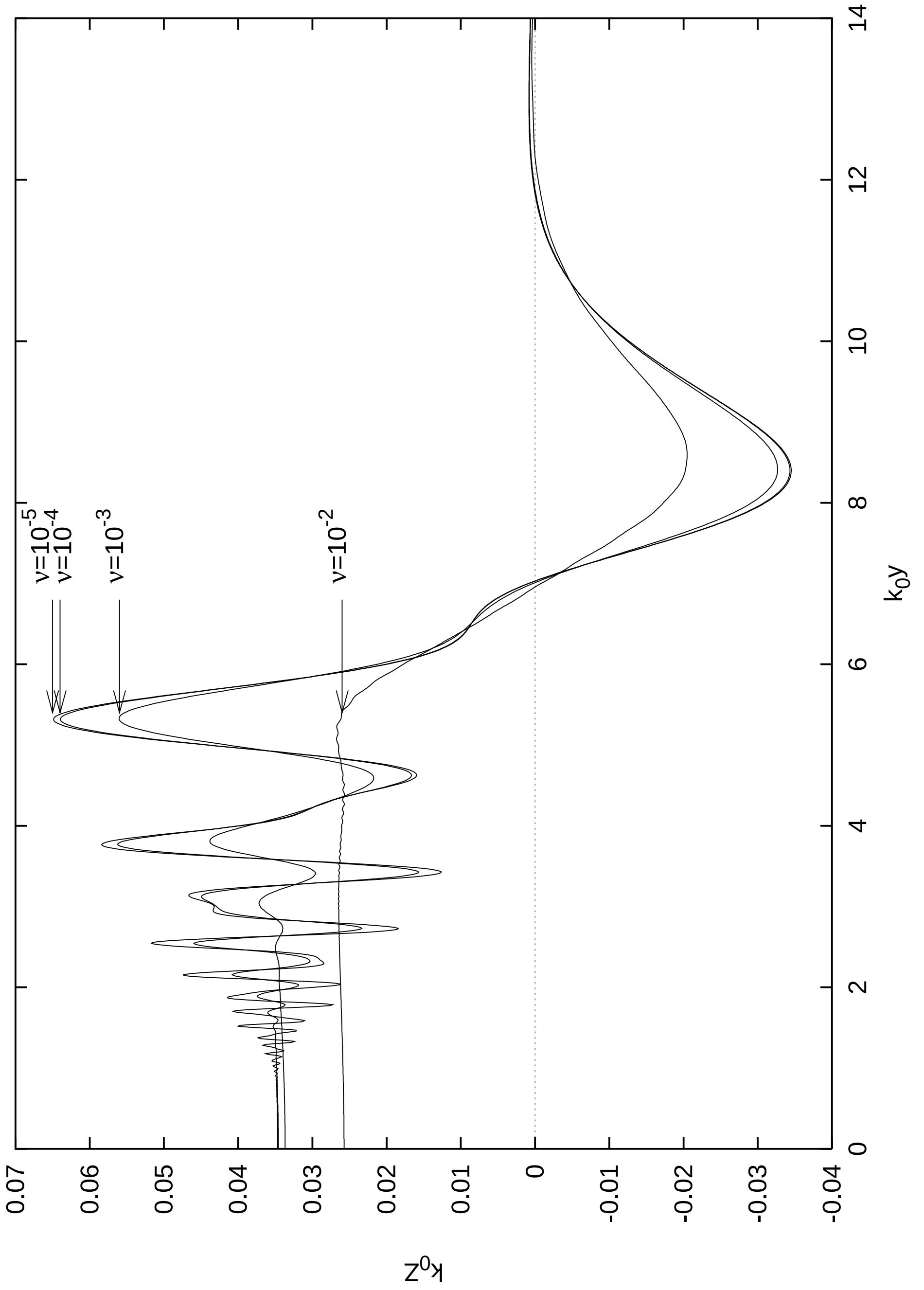

Figure 1: Lateral cut of free surface behind destroyer hull at 30 knots. Plots for various values of viscosity.

tor V as programmed in `SWPE3.0` is quite reasonable throughout the wave field. That is, the fact that it neither depends on y, nor takes account of non-uniformity of eddy viscosity, is not important. In a rough sense, this is because y never gets large enough compared to x for any waves within the Kelvin angle $|y| < 0.35x$. Indeed, y is particularly small for the most divergent waves which propagate from the centreline with low velocity. Hence although there are variations with y of the effective damping rates for a given viscosity, and there may also be variations of the viscosity itself with y (not investigated here), the effect of these variations on the actual wave pattern is likely to be negligible relative to the much larger (though still small!) simple exponential decay with respect to x, as already programmed in `SWPE3.0` and later versions.

As far as neglect of viscosity variation with respect to y is concerned, the choice one should make for the actual viscosity coefficient ν to use in `SWPE` is that which is appropriate for $y = 0$. That is, we recommend setting the constant ν to be the eddy viscosity in the immediate wake along the track of the ship. This is because the main effect of the viscous factor is felt for small y, and even if the (uniform) value assumed is far too large when y is not small (but still within the Kelvin angle), this is of little consequence to the wave pattern computations.

Literature survey and viscosity value

We have spent a considerable amount of time and effort seeking further published information on viscous effects on ship waves, with limited success. We have found only one reference (Hatano et al 1977) where a measured value of eddy viscosity in a ship wake is quoted, namely $\nu = 0.0002\ \mathrm{m^2s^{-1}}$. This was for a rather small model at low Froude number, and somewhat higher values might be appropriate for larger models or full scale.

Otherwise, even in review articles like Stern et al (2000) which encompass both experiments and computation, the eddy viscosity is mentioned but an actual numerical value is never quoted. For example (p. 29), "…give *reasonable* eddy viscosity values …" and "…prevent *too large* values of eddy viscosity …"; it is hard to see how such judgements can be made without some criterion for reasonableness! It would be valuable to have a measure of a typical eddy viscosity encountered in the process of a CFD computation using more complex turbulence models, but such values seem not to be revealed, for reasons about which we could speculate.

There have been a few theoretical approaches, and a very recent paper by Zilman and Miloh (2001) seems valuable not only for the technical content but also for adding references to earlier work such as Cumberbatch (1965), Brard (1970), Peregrine (1971), Lurye (1973) and Milgram et al (1993). Some of this work is similar to Tuck (1974), and of course work on viscous damping of water waves goes back to Lamb (1932) (pp. 623–628) and even to Basset (1888) (pp. 520–522). Recent papers on viscous and/or turbulence effects on surface water waves include Chan and Chwang (1997), Drennan et al (1997) and Teixera and Belcher (2001). None of the above references quote a viscosity other than the molecular value $\nu = 10^{-6}$ $\mathrm{m^2s^{-1}}$, which is far too small to be significant for normal ship waves.

There seems to be no other published information on just what is a typical value of an eddy viscosity for a ship wake, but we reiterate that a `SWPE3.0` viscosity of about $\nu = 0.005$ $\mathrm{m^2s^{-1}}$ gave reasonable agreement with longitudinal cut results (including apparent damping of transverse waves) from one published experiment, namely the Wake-Off test (Lindemuth et al 1990). A value of $\nu = 0.0025$ $\mathrm{m^2s^{-1}}$ in the factor V_3 as programmed in the current version `SWPE6.0` would produce the same viscous decay on the centreline as $\nu = 0.005$ $\mathrm{m^2s^{-1}}$ in `SWPE3.0`. Nevertheless, we now feel that this is rather too high a viscosity for routine use, and envisage eddy viscosities in the range 0.0001–0.0010, preferably at the low end of that range, close to the value 0.0002 quoted by Hatano et al (1972). That is, in the present report we take a somewhat more conservative view, in which we prefer to leave the transverse waves alone as far as possible, while allowing viscosity to damp out the shortest diverging waves close to $y = 0$. Hence our recommended viscosity for routine use in `SWPE6.0` is $\nu = 0.0002$ $\mathrm{m^2s^{-1}}$.

Appendix 1: Rayleigh-type viscosity

One procedure for introducing viscous effects into water wave theory is to add a dissipative term to the linearised free-surface condition. Thus for unsteady waves in a fixed frame of reference, the usual condition is

$$g\phi_z + \phi_{tt} = 0$$

which permits undamped sinusoidal waves in water of infinite depth of the form

$$\phi = \exp[ikx - i\sigma t + kz] \tag{15}$$

if

$$\sigma = \sqrt{gk}\ .$$

We now suggest the modified free-surface condition

$$g\phi_z + \phi_{tt} + 4\nu\phi_{tzz} = 0 \tag{16}$$

where ν is the (small) kinematic viscosity. That this term is dissipative can be seen by substituting the wave form (15) for ϕ, giving

$$gk - \sigma^2 - 4i\nu\sigma k^2 = 0\ ,$$

a quadratic equation which has the solution for σ (assuming positive real part)

$$\sigma = \sqrt{gk - 4\nu^2 k^4} - 2i\nu k^2$$

The imaginary part of σ then agrees with the above Lamb dissipative factor.

Now in a frame of reference moving to the left at speed U, the free-surface condition (16) becomes

$$g\phi_z + U^2\phi_{xx} + 4\nu U\phi_{xzz} = 0 \tag{17}$$

The rather better-known Rayleigh artificial viscosity is similar to (17), but without the two z-derivatives, i.e. the extra term just involves one x-derivative. On the other hand, this type of free-surface condition is only one of many possibilities in surface rheology; for example, a condition with three z derivatives rather than two was used in Tuck (1974) for a thin surface layer of viscous fluid. However, the actual form (16) chosen here has the advantage of agreeing with Lamb, who allowed for true viscous stresses in the fluid.

Now we can repeat the usual derivation of the thin-ship wave pattern with the new steady free-surface condition (17). Only the barest details are necessary. Thus the solution is the integral over both wavenumber k and direction θ of plane waves

$$\exp[ik(x\cos\theta + y\sin\theta) + kz] \tag{18}$$

The amplitude of this wave contains a simple pole at the value of k where the free-surface condition (17) holds for that factor, i.e. where

$$gk - U^2 k^2 \cos^2\theta + 4\nu U(ik\cos\theta)k^2 = 0\ .$$

The usual solution ($\nu = 0$) is

$$k = k_0 \sec^2\theta$$

and a first correction for small ν is

$$k = k_0 \sec^2\theta + \frac{4i\nu}{U} k_0^2 \sec^5\theta\ . \tag{19}$$

Now in the far field, the solution is dominated by the residue at this pole, so there is a term involving the factor (18) but with k given by (19), i.e. the part of this factor depending on x and y is

$$\exp\left[i(k_0\sec^2\theta + \frac{4i\nu}{U}k_0^2\sec^5\theta)(x\cos\theta + y\sin\theta)\right]$$

which is the usual wave factor

$$\exp\left[ik_0\sec^2\theta(x\cos\theta + y\sin\theta)\right]$$

times the dissipation factor

$$V = \exp\left[-\frac{4\nu}{U}k_0^2\sec^5\theta(x\cos\theta + y\sin\theta)\right]\ . \tag{20}$$

This is the damping factor V_3 as given in (11).

Appendix 2: Stationary phase arguments

In the far field, the wave pattern is given by an integral with respect to θ with a phase factor

$$\exp\left[ik_0\sec^2\theta(x\cos\theta + y\sin\theta)\right] = \exp\left[ik_0 r\sec^2\theta\cos(\theta - \beta)\right]$$

if $x = r\cos\beta$ and $y = r\sin\beta$. Hence for large $k_0 r$, this integral is dominated by contributions from the neighbourhood of θ values such that

$$\frac{d}{d\theta}\sec^2\theta\cos(\theta - \beta) = 0$$

which leads to $\tan(\theta - \beta) = 2\tan\theta$. This equation can also be written as a quadratic equation for $\tan\theta$, given $\tan\beta$, and therefore has two solutions, which are the transverse waves

$$\tan\theta_T = \frac{-1 + \sqrt{1 - 8\tan^2\beta}}{4\tan\beta}$$

and the diverging waves

$$\tan\theta_D = \frac{-1 - \sqrt{1 - 8\tan^2\beta}}{4\tan\beta}$$

inside the Kelvin angle $\tan^2\beta < 1/8$.

We can now immediately see that the choices V_3 and V_2 give identical dissipation rates subject to this stationary phase approximation, since

$$\begin{aligned} x - y\cot\theta &= r\frac{\sin(\theta-\beta)}{\sin\theta} \\ &= r\frac{\cos(\theta-\beta)\tan(\theta-\beta)}{\sin\theta} \\ &= r\frac{\cos(\theta-\beta)\ 2\tan\theta}{\sin\theta} \\ &= 2r\frac{\cos(\theta-\beta)}{\cos\theta} \\ &= 2(x + y\tan\theta)\ , \end{aligned}$$

so $V_2 = V_3$. This is a remarkable result, considering the very different derivations.

Note that when $\theta > 0$, the stationary points have $\beta < 0$, so $y < 0$. Then the term $-y/\tan\theta$ in t_2 increases the damping whereas the term $+y\tan\theta$ in t_3 decreases it relative to that when $y = 0$; the factor of 2 in t_3 seems to be precisely what is required to compensate for this decrease. On the other hand, this means that t_3 overdamps at $y = 0$ relative to $t_0 = x/U$.

This feature can be illustrated by noting that

$$\frac{t_3}{t_0} = \frac{2 + 2\tan^2\theta}{1 + 2\tan^2\theta}$$

is close to 1 at $\theta = \pi/2$, but close to 2 at $\theta = 0$, whereas the halved value

$$\frac{t_4}{t_0} = \frac{1 + \tan^2\theta}{1 + 2\tan^2\theta}$$

is close to 1 at $\theta = 0$, but close to $1/2$ at $\theta = \pi/2$. Since t is multiplied by $\sec^4\theta$ in the exponent of the viscous damping factor V, it is probably of less consequence to underestimate t at $\theta = \pi/2$ (extreme diverging waves close to the track that are already highly damped) than to overestimate it for the transverse waves at $\theta = 0$.

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